

ON EMBEDDINGS OF AFFINOR-SEMISYMMETRIC SPACES INTO FLAT SPACES

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Consider a pseudo-Riemannian space V_n of class one with a metric tensor $g_{ij}(x)$, in which an affinator $F_i^h(x)$ is defined. The tensor characteristics of class one spaces take the form

$$R_{hijk} = e(b_{hk}b_{ij} - b_{hj}b_{ik}), \quad b_{ij,k} - b_{ik,j} = 0, \quad (1)$$

where b_{ij} is the second fundamental form of the space V_n , and $e = \pm 1$. It is known [1] that spaces of the first class cannot be Ricci-flat.

Suppose that the affinator structure defined in this space is antisymmetric, semisymmetric [2], and almost complex, i.e., such that it satisfies

$$F_{ij,[kl]} = 0, \quad F_{\cdot\alpha}^h F_{\cdot i}^\alpha = -\delta_i^h, \quad F_{(ij)} = 0. \quad (2)$$

Spaces for which the first condition holds will be called affinator-semisymmetric. In spaces with an affinator structure, the operation of conjugation is defined. We write the integrability conditions of an affinator, taking into account the Ricci identity, in terms of the conjugation operation:

$$R_{\bar{i}jkl} = -R_{i\bar{j}kl}. \quad (3)$$

Contracting with the affinator with respect to the index j , taking into account the almost complex structure of the space, and changing the indices in terms of the conjugation operation, we obtain

$$R_{i\bar{j}\bar{k}l} = R_{ijkl}. \quad (4)$$

It follows that the integrability conditions of an arbitrary tensor $A_{i_1 i_2 \dots i_q}$, taking into account the Ricci identity, satisfy

$$A_{i_1 i_2 \dots i_q, [\bar{k}l]} = A_{i_1 i_2 \dots i_q, [kl]}, \quad (5)$$

hence for the curvature tensor we have

$$R_{hijk, [\bar{p}]} = R_{hijk, [lp]}, \quad (6)$$

and, therefore,

$$R_{ij} = 0, \quad (7)$$

which contradicts the properties of spaces of the class one. We have proven the following

Theorem 1. *There are no affinator-semisymmetric spaces of the class one.*

Thus, pseudo-Riemannian spaces of the first class endowed with an elliptic affinator-semisymmetric structure of the second degree have been investigated. Algebraic analysis of the second quadratic form of these spaces shows that the existence of such a geometric structure is impossible in these spaces.

Acknowledgements

[1] Kiosak V., Latysh O., Kuzmich V., Split curvature, *Proc. Int. Geom. Center* **17** (2024), no. 3, 232–243.

[2] Soloviov A., Mappings of pseudo-Riemannian spaces with a semisymmetric affinator structure, *Mat. Doslid. Mekh.* **29** (2024), no. 1(43), 14–33.