International Conference of Young Mathematicians
The Institute of Mathematics of the National Academy of Sciences of Ukraine
June 3–5, 2026, Kyiv, Ukraine

On d-characteristics of the high-order boundary-value problems in spaces of differentiable functions

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Let integers $m, l, r \geq 1$ and $n \geq 0$ be arbitrarily chosen. We present some results on d-characteristics (i.e., the dimensions of the problem kernel and co-kernel) of the following linear boundary-value problem for a system of m high-order differential equations on the finite interval $(a, b) \subset \mathbb{R}$

$$(Ly)(t) := y^{(r)}(t) + \sum_{j=1}^{r} A_{r-j}(t) y^{(r-j)}(t) = f(t), \quad t \in (a, b), \tag{1}$$

$$By = c. \tag{2}$$

Here, the matrix function $A_{r-j} \in (C^{(n)})^{m\times m}$ with $1 \leq j \leq r$, the vector function $f \in (C^{(n)})^m$, the numerical vector $c \in \mathbb{C}^l$ and the linear continuous operator $B\colon (C^{(n+r)})^m \to \mathbb{C}^l$ are arbitrarily chosen.

The boundary condition (2) gives l scalar boundary conditions for a system of m first order differential equations. For $l > rm$, this boundary condition is *overdetermined* and, for $l < rm$, it is *underdetermined* with respect to the differential system (1).

Using the linear continuous operator

$$(L, B)\colon (C^{(n+r)})^m \to (C^{(n)})^m \times \mathbb{C}^l, \tag{3}$$

we rewrite the boundary-value problem (1), (2) in the form of the equation $(L, B)y = (f, c)$.

Theorem 1. *Operator* (3) *is Fredholm with index* $rm - l$.

For every number $i \in \{1, \dots, r\}$, consider the following family of matrix Cauchy problems

$$Y_i^{(r)}(t) + \sum_{j=1}^{r} A_{r-j}(t) Y_i^{(r-j)}(t) = O_m, \quad t \in (a, b),$$
$$Y_i^{(j-1)}(a) = \delta_{i,j} I_m, \quad j \in \{1, \dots, r\},$$

where $Y_i(\cdot)$ denotes an unknown $m \times m$ matrix function, O_m and I_m denote, respectively, the null and identity $m \times m$ matrices, and $\delta_{i,j}$ is the Kronecker delta.

Let $[BY_i]$ denote a complex numerical $l \times m$ matrix, in which each j-th column is the result of the action of the operator B on the j-th column of the matrix function $Y_i(\cdot)$.

Definition 1. The numerical $l \times rm$ matrix

$$M(L, B) := \big([BY_1], \dots, [BY_r] \big) \in \mathbb{C}^{l \times rm},$$

which consists of r rectangular block columns $[BY_i] \in \mathbb{C}^{m\times l}$, is called *the characteristic matrix* of the boundary-value problem (1), (2).

Theorem 2. *For d-characteristics of the Fredholm operator* (3), *the following relations*

$$\dim\ker(L,B) = \dim\ker M(L,B),$$
$$\dim\operatorname{coker}(L,B) = \dim\operatorname{coker} M(L,B)$$

are true.

Corollary 1. *Operator* (3) *is invertible* (*i.e., a topological isomorphism*) *if and only if* $l = rm$ *and the square matrix* $M(L,B)$ *is nonsingular.*

These results were published in [1]. Similar first order problem was considered in [2].

The author was supported by a scholarship of the President of Ukraine for young scientists, and by the grant of the Simons Foundation (SFI-PD-Ukraine-00014586, V.O.S.).

[1] Soldatov V., Solvability of linear boundary-value problems for ordinary high-order differential systems in spaces of continuously differentiable functions, *Ukr. Mat. Zhurn.* (accepted for publication) (2026), 16 pp.

[2] Soldatov V., Solvability of Linear Boundary-Value Problems for Ordinary Differential Systems in the Space C^n, *Ukrainian Math. J.* **77** (2025), No 3, 360–368.