# EQUIVALENCE OF THREE TOPOLOGIES IN SPACES OF LAPLACE–STIELTJES INTEGRALS

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**Abstract**

We study spaces generated by Laplace–Stieltjes integrals of the form

$$I(\sigma) = \int_1^\infty f(x)e^{x\sigma}\, dF(x), \quad \sigma \in \mathbb{R},$$

where $F$ is a non-negative, nondecreasing, unbounded, right-continuous function on $(1, +\infty)$ and $f$ is a real-valued function. Such integrals and their asymptotic behavior have been extensively studied in the literature (see, e.g., [1]).

We consider a class $U(F)$ of functions whose absolute values satisfy a regular variation condition with respect to $F$, and investigate the corresponding space $LS(U(F))$ of Laplace–Stieltjes integrals. Three natural topologies are introduced on $LS(U(F))$: a metric topology generated by a countable system of norms, and two additional topologies defined via paranorms associated with pointwise growth conditions (see [2]).

Under a mild condition on $F$ ensuring sufficient decay of exponential integrals, we prove that these topologies are equivalent. This result provides a unified framework for studying convergence in $LS(U(F))$ and implies, in particular, uniform convergence on compact intervals.

Furthermore, we establish completeness of the space under these topologies and show that it forms a Fréchet space. Additional structural properties, including the Montel property, are also obtained, extending earlier results on functional spaces of Laplace–Stieltjes integrals [3].

## References

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