HÖLDER CONTINUITY OF SOLUTIONS TO DOUBLY NONLINEAR PARABOLIC EQUATIONS IN THE MIXED SINGULAR AND DEGENERATE RANGE

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Let Ω be a domain in $\mathbb{R}^N$, $T > 0$, and denote $\Omega_T := \Omega \times (0,T)$. We study nonnegative (sub/super)-solutions to the equation

$$u_t - \operatorname{div}\mathbf{A}(x,t,u,Du) = 0, \qquad (x,t) \in \Omega_T. \tag{1}$$

Throughout the paper we suppose that the functions $\mathbf{A} = (A_1, \cdots, A_N) : \Omega_T \times \mathbb{R}_+ \times \mathbb{R}^N \to \mathbb{R}^N$ are Carathéodory, i.e. such that $\mathbf{A}(\cdot,\cdot,u,\xi)$ are Lebesgue measurable for all $u \in \mathbb{R}_+$, $\xi \in \mathbb{R}^N$, and $\mathbf{A}(x,t,\cdot,\cdot)$ are continuous for almost all $(x,t) \in \Omega_T$. We also assume that the following structure conditions are satisfied

$$\begin{cases} \mathbf{A}(x,t,u,Du)Du^{m^-} \geqslant K_1\, u^{(m-m^-)(p-1)}|Du^{m^-}|^p, \\ |\mathbf{A}(x,t,u,Du)| \leqslant K_2\, u^{(m-m^-)(p-1)}|Du^{m^-}|^{p-1}, \end{cases} \tag{2}$$

where $m > 0$, $p > 1$, K_1, K_2 are positive constants and $m^- := \min(1,m)$.

The prototype of equation (1) is the doubly nonlinear parabolic equation

$$u_t - \operatorname{div}\big(|Du^m|^{p-2}Du^m\big) = 0, \qquad p > 1, \quad m > 0.$$

Definition 1. We say that a function u is a nonnegative, locally bounded, local weak (sub) super-solution to (1) if

$$0 \leqslant u \in C_{\mathrm{loc}}(0,T; L^{1+m^-}_{\mathrm{loc}}(\Omega)), \qquad u^{m^-} \in L^p_{\mathrm{loc}}(0,T; W^{1,p}_{\mathrm{loc}}(\Omega)), \qquad u \in L^\infty_{loc}(\Omega_T),$$

and for any compact set $E \subset \Omega$ and every subinterval $[t_1,t_2] \subset (0,T]$ there holds

$$\int\limits_E u\zeta\, dx\Big|_{t_1}^{t_2} + \int\limits_{t_1}^{t_2}\int\limits_E \{-u\zeta_\tau + \mathbf{A}(x,\tau,u,Du)\, D\zeta\}\, dxd\tau \leqslant (\geqslant)\, 0,$$

for any testing functions $\zeta \in W^{1,1+\frac{1}{m^-}}(0,T; L^{1+\frac{1}{m^-}}(E)) \cap L^p(0,T; W^{1,p}_0(E))$, $\zeta \geqslant 0$.

Remark 1. In order to simplify the presentation, we have decided to give a definition of solution that already encodes the nonnegativity of u and its local boundedness, besides a certain integrability of the gradient of small powers of u. For these topics, we refer to [1] and references therein.

Our main result reads as follows. Let the main degeneracy exponent be

$$\lambda = m(p-1)\,.$$

Theorem 1. [2] *Let u be a nonnegative, local weak solution to* (1) – (2) *in Ω_T and assume also that one of the following conditions holds*

$$\lambda < 1 \quad and \quad p + N(\lambda - 1) > 0,$$

or

$$\lambda > 1 \quad and \quad \frac{2N}{N+1} < p < 2,$$

or

$$\lambda > 1 \quad and \quad p > 2,$$

then u is locally Hölder continuous in Ω_T.

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[2] Ciani S., Henriques E., Savchenko M., Skrypnik I. and Yevgenieva Ye., Regularity for Doubly Nonlinear Equations in the Mixed Regime, *Advances in Calculus of Variations*, (2026), 30 pp., arXiv: 2602.09906.