Sensor fusion for mobile robot localization via Kalman filtering

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Mobile-robot navigation on a plane requires accurate knowledge of the state – position (x, y) and orientation θ – a canonical challenge for autonomous vehicles, warehouse automation, planetary exploration, and assistive devices [1]. The individual sensing modalities are complementary: wheel odometry from encoder integration yields high-frequency relative motion but accumulates unbounded error from noise and systematic wheel defects, while GPS supplies absolute positions at a low rate with significant noise. Optimal fusion of the two is therefore essential.

Consider a differential-drive (two-wheeled) mobile robot with state $\mathbf{x}_k = [x_k, y_k, \theta_k]^\top \in \mathbb{R}^3$. Two wheel encoders feed standard differential-drive kinematics to produce body-frame linear and angular velocities, integrated over the sample interval Δt to yield a noisy odometry increment $\Delta \mathbf{z}_k^{\text{odom}} \in \mathbb{R}^3$. GPS provides an absolute position measurement (a *fix*) $\mathbf{z}_k^{\text{gps}} \in \mathbb{R}^2$. The discrete linear model is

$$\mathbf{x}_k = \mathbf{x}_{k-1} + \Delta \mathbf{z}_k^{\text{odom}} + \mathbf{w}_k, \qquad \mathbf{z}_k^{\text{gps}} = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k, \tag{1}$$

where $\mathbf{H} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ extracts the planar position $(x_k, y_k)^\top$ from $\mathbf{x}_k$, $\mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$ is the process noise, and $\mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$ is the GPS measurement noise. The problem is to recursively compute, from $\Delta \mathbf{z}_k^{\text{odom}}$ and $\mathbf{z}_k^{\text{gps}}$, the minimum-variance estimate $\hat{\mathbf{x}}_{k|k}$ together with its posterior error covariance $\mathbf{P}_{k|k} = \operatorname{cov}[\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}]$ [2], where the subscript $k|k$ denotes a quantity at time k conditioned on all measurements up to and including time k; $\mathbf{P}_{k|k}$ thus quantifies the remaining uncertainty in the estimate.

The Kalman filter [3] is the minimum-variance recursive estimator for the linear-Gaussian model (1). It alternates two steps: between consecutive GPS fixes it *predicts*, carrying the estimate forward by the odometry increment as its uncertainty grows,

$$\hat{\mathbf{x}}_{k|k-1} = \hat{\mathbf{x}}_{k-1|k-1} + \Delta \mathbf{z}_k^{\text{odom}}, \qquad \mathbf{P}_{k|k-1} = \mathbf{P}_{k-1|k-1} + \mathbf{Q}, \tag{2}$$

and at each fix it *corrects*, pulling the estimate toward the measurement as its uncertainty shrinks,

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\big(\mathbf{z}_k^{\text{gps}} - \mathbf{H}\hat{\mathbf{x}}_{k|k-1}\big), \qquad \mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k\mathbf{H})\mathbf{P}_{k|k-1}, \tag{3}$$

where the *innovation* $\mathbf{z}_k^{\text{gps}} - \mathbf{H}\hat{\mathbf{x}}_{k|k-1}$ is the gap between the fix and its prediction, weighted by the Kalman gain

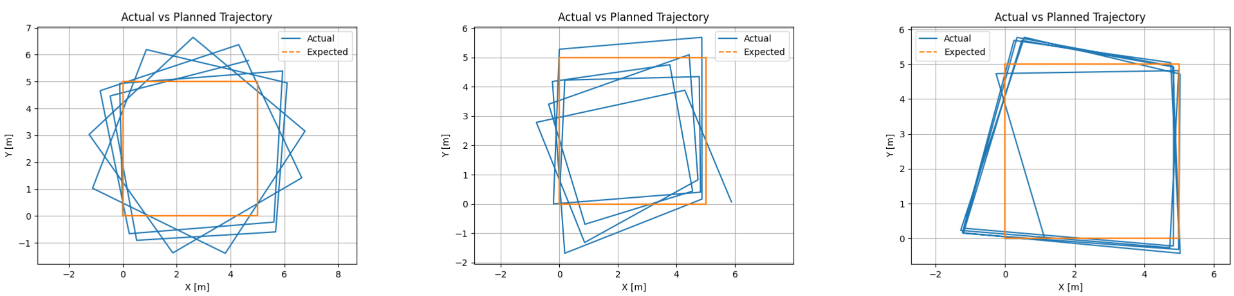
$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}^\top\big(\mathbf{H}\mathbf{P}_{k|k-1}\mathbf{H}^\top + \mathbf{R}\big)^{-1}, \tag{4}$$

which is the unique linear weighting that minimizes $\operatorname{tr}(\mathbf{P}_{k|k})$ – the sum of posterior variances of the three state coordinates, equivalent to the expected squared estimation error [2, 4]. Structurally, the gain $\mathbf{K}_k$ (4) is the matrix analogue of the scalar regression slope (covariance divided by variance). It maps the innovation through its two factors: the state-measurement cross-covariance $\mathbf{P}_{k|k-1}\mathbf{H}^\top$ and the inverse innovation covariance $(\mathbf{H}\mathbf{P}_{k|k-1}\mathbf{H}^\top + \mathbf{R})^{-1}$. When the prediction (2) is uncertain (large $\mathbf{P}_{k|k-1}$) the filter trusts the GPS, and when the GPS is noisy (large $\mathbf{R}$) it trusts the prediction.

The estimator is evaluated in the Gazebo simulator on a differential-drive robot driven through five laps of a 5×5 m square (100 m total), with a 0.3% wheel-radius defect and GPS noise $\sigma_{\text{gps}} = 0.5$ m. Accuracy against the simulator ground truth $\mathbf{x}_k^{\text{truth}}$ is measured by the root-mean-square error (RMSE), the typical position error over the whole run, and the final-position error (FPE) at the last step,

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left\|\hat{\mathbf{x}}_k - \mathbf{x}_k^{\text{truth}}\right\|^2}, \qquad \text{FPE} = \left\|\hat{\mathbf{x}}_N - \mathbf{x}_N^{\text{truth}}\right\|. \tag{5}$$

By both metrics (5) the Kalman filter substantially outperformed both individual sensors, producing a smooth, globally consistent trajectory: odometry alone (Fig. 1a), RMSE = 3.82 m, FPE = 6.54 m; GPS alone (Fig. 1b), RMSE = 1.05 m, FPE = 1.28 m; fusion (Fig. 1c), RMSE = 0.33 m, FPE = 0.52 m – a 91.4% RMSE reduction over odometry and 68.5% over GPS. Thus Kalman filtering, even with a linear approximation of the nonlinear kinematics, is practically essential for reliable mobile-robot localization.



(a) Odometry only (b) GPS only (c) Kalman filter

Figure 1: Sensor fusion drives the estimated trajectory (blue) onto the expected square (orange). (a) Odometry alone: the wheel-radius defect produces unbounded drift, rotating and translating the estimated square off the true path. (b) GPS alone: absolute fixes bound the long-term error but noise yields a jagged trace. (c) Kalman filter: the prediction (2) follows odometry between fixes, and each GPS fix applies the correction (3), snapping the estimate back toward the truth.

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