Representations of Munn algebras and related semigroups

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It is a joint work with Yu. Drozd. The results are published in [2].

Let F be a finite dimensional skewfield over a field $\Bbbk$, and let $m, n, r \in \mathbb{N}$. The *Munn algebra* $\mathbb{M}(F, m, n, r)$ is defined as the ring of $(n+r) \times (m+r)$ matrices over F with the multiplication $A \cdot B = A\mu B$, where μ is an $(m+r) \times (n+r)$ matrix of rank r [1,2].

Let $\mathbb{M} = \prod_{i=1}^{s} \mathbb{M}(F_i, m_i, n_i, r_i)$, $d_i = \dim_{\Bbbk} F_i$, and $\mathfrak{T} = \{(d_i, m_i, n_i) \mid (m_i, n_i) \neq (0,0)\}$.

Let $\mathfrak{T} = \mathfrak{T}^- \cup \mathfrak{T}^+ \cup \mathfrak{T}'$, where

$$\mathfrak{T}^- = \{(d_i, 1, 0) \mid 1 \leq i \leq q\},$$
$$\mathfrak{T}^+ = \{(d_j, 0, 1) \mid q+1 \leq j \leq s\}.$$

Set $S^- = \sum_{i=1}^{q} d_i, \qquad S^+ = \sum_{j=q+1}^{s} d_j, \qquad S = S^- + S^+.$

Theorem 1. *1. $\mathbb{M}$ is representation finite if and only if*

(a) either $\mathfrak{T}' = \emptyset$ and $\max\{S^-, S^+\} \leq 3$;

(b) or $\mathfrak{T}' = \{(1,1,1)\}$, $S \leq 3$, and $\max\{S^-, S^+\} \leq 2$.

2. $\mathbb{M}$ is representation tame if and only if

(a) either $\mathfrak{T}^+ = \mathfrak{T}^- = \emptyset$ and $\mathfrak{T}'$ is one of the sets

$$\{(1,1,1),(1,1,1)\}, \quad \{(2,1,1)\}, \quad \{(1,2,0)\}, \quad \{(1,0,2)\};$$

(b) or $\mathfrak{T}' = \emptyset$ and $\max\{S^-, S^+\} = 4$;

(c) or $\mathfrak{T}' = \{(1,1,1)\}$ and $S^- = S^+ = 2$.

3. In all other cases $\mathbb{M}$ is representation wild.

The algebraically closed case (where all $d_i = 1$) coincides with the result of Ponizovskii [3, No. 5].

Using this result we establish the representation type of finite Rees matrix semigroups [1], in particular, 0-simple semigroups, and their mutually annihilating unions in the case when the characteristic of the field $\Bbbk$ does not divide the orders of the involved groups.

We devote this work to the memory of I. S. Ponizovskii.

[1] Clifford A. H., Preston G. B., *The algebraic theory of semigroups. Vol. I.*, American Mathematical Society, Providence, RI, 1961.

[2] Drozd Yu. A., Plakosh A. I., Representations of Munn algebras and related semigroups, *Communications in Algebra* **51** (2023), no. 3, 930–938.

[3] Ponizovskii I. S., On the finiteness of type of a semigroup algebra of a finite fully prime semigroup, *J. Sov. Math.* **3** (1975), 700–709.