On topological properties of weakly two-convex sets in three-dimensional Euclidean space

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We consider the properties of generalized convex sets in real three-dimensional Euclidean space $\mathbb{R}^3$, known as weakly 2-convex. An open set in $\mathbb{R}^3$ is called *weakly* 2*-convex* if through every boundary point of the set, there passes a plane that does not intersect the given set. A closed set in $\mathbb{R}^3$ is called *weakly* 2*-convex* if it is approximated from the outside by a family of open weakly 2-convex sets. A point in the complement of a set in $\mathbb{R}^3$ (with respect to the entire space) is a 2*-nonconvexity point* of the set if every plane passing through the point intersects the set.

It is proved that the 2-nonconvexity-point set corresponding to an open weakly 2-convex set in $\mathbb{R}^3$ is itself open and weakly 2-convex [1], that the non-empty interior of a solid closed weakly 2-convex set is weakly 2-convex [2]. A number of convexity properties of the 2-nonconvexity-point sets corresponding to weakly 2-convex sets in $\mathbb{R}^3$ are also investigated. In particular, It is shown that for any convex polyhedron there exists an open weakly 2-convex set such that its 2-nonconvexity-point set coincides with the polyhedron interior. However, there exists an open weakly 2-convex set such that its open bounded convex 2-nonconvexity-point set differs from the interior of a convex polyhedron [2]. Nevertheless, not every open convex set can occur as a connected component of the 2-nonconvexity-point set corresponding to some open weakly 2-convex set [1].

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[1] Osipchuk T., On weakly $(n-1)$-convex sets in low-dimensional Euclidean spaces. *Ukrainian Mathematical Journal*, **78** (2026) (accepted for publication).

[2] Osipchuk T., On weakly two-convex sets in three-dimensional Euclidean space. *Proceedings of the International Geometry Center*, **19** (2026), 2 (accepted for publication).