Problem with integral condition for homogeneous system of partial differential equations of third order in Sobolev space

G. Kuduk[1]

[1]Faculty of Mathematics and Natural Sciences University of Rzeszow,
Graduate of University of Rzeszow

gkuduk@onet.eu

In the strip $\Omega(T) : \{(t,x) \in \mathbb{R}^{n+1} : t \in ([T_1, T_2 \cup [T_3, T_4]), x \in \mathbb{R}^n\}$ we consider nonlocal problem with integral conditions

$$L\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right)\overrightarrow{U}(t,x) = \frac{\partial^3 \overrightarrow{U}(t,x)}{\partial t^3} + A_1 \frac{\partial^3 \overrightarrow{U}(t,x)}{\partial t^2 \partial x} + A_2 \frac{\partial^3 \overrightarrow{U}(t,x)}{\partial t \partial x^2} + A_3 \frac{\partial^3 \overrightarrow{U}(t,x)}{\partial x^3} = \overrightarrow{0}, \quad (1)$$

$$\int\limits_{T_1}^{T_2} t^k \overrightarrow{U}(t,x)dt + \int\limits_{T_3}^{T_4} t^k \overrightarrow{U}(t,x)dt = \overrightarrow{\varphi}_i(x), \quad k = \{0,1,2,3\}, \quad (2)$$

where $\overrightarrow{U}(t,x) = col\,(U^1(t,x), U^2(t,x), U^3(t,x))$, $\overrightarrow{\varphi}_i(x) = col\,(\varphi_i^1, \varphi_i^2, \varphi_i^3)$, $i = \{1,2,3\}$, A_1, A_2, A_3 are squert matrix 3×3. Let $H_\alpha, \alpha > 0$ be a Sobolew space with norm [2].

$$\|\varphi(x); H_\alpha\| = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{+\infty} (1+|\xi|)^{2\alpha} |\widetilde{\varphi}(\xi)|^2 d\xi} < \infty,$$

where $\widetilde{\varphi}(\xi)$ is a Fourier transformation of the function $\varphi(x)$. $\overline{H}_\alpha$ is a vector space of the function $\varphi(x)$, with norm

$$\|\varphi(x), \overline{H}_\alpha\| = \max \|\varphi^j(x); H_\alpha\|.$$

Theorem 1. *Let conditions occur, let $\Delta(\xi) \neq 0$ for everyone $\xi \in \mathbb{R} \setminus \{0\}$. If $\overrightarrow{\varphi}_j \in \overline{H}^3_{\alpha_1}$, $\alpha_1 \geqslant \alpha_2 + 3\,(C_3^2 + 1)$, $\alpha_2 > 1$, $j = \{1, ..., n\}$, then the space $C^3([T_1, T_2] \cup [T_3, T_4]), \overline{H}^3_\alpha)$ exists and unique solution $\overrightarrow{U}(t,x)$ of the problem (1)–(2), which still is depending on vector-function $\overrightarrow{\varphi}_j$, $j = \{1,2,3\}$.*

Solution is the represented in the form

$$\overrightarrow{U}(t,x) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{+\infty} e^{ix\xi} \sum_{j,q=1}^{9} \frac{\Delta_{j,q}(\xi)}{\Delta(\xi)} e^{\lambda_q \xi} \overrightarrow{h}_q \psi_j(\xi) d\xi, \quad (3)$$

where $\Delta_{j,q}(\xi)$, $j,q = \{1, \dots, 9\}$, $\xi \neq 0$, algebraic complement the standing element j this poem and q for this column of indicator $\Delta(\xi)$,

$$\mathrm{col}(\psi_1(\xi), \dots, \psi_9(\xi)) = \mathrm{col}(\tilde{\varphi}_1^1(\xi), \dots, \tilde{\varphi}_1^3(\xi); \dots; \tilde{\varphi}_3^1(\xi), \dots, \tilde{\varphi}_3^3(\xi)),$$

and $\tilde{\varphi}_j^q(\xi)$ is a Fourier transformation of the function $\varphi_j^q(x)$, $j = \{1,2,3\}$, $q = \{1,2,3\}$. This result continues the research of work [1,3–6].

[1] Kalenyuk P. I, Nytrebych Z. M., Kuduk G., Symotyuk M. M., Problem with integral conditions for system equations of second order, *Proceedings of 13-th. Scientific Conference dedicated the 125-th anniversary of Stefan Banach (Lviv, Ukraine, March 30–31, 2017)*, Lviv Polytechnic National University, 42–43.

[2] Kalenyuk P. I, Nytrebych Z. M., Kuduk G., Symotyuk M. M., Integral problem for partial differential equation of second order in unboudud layer, *Bukovinian Mathematical Journal* **4** (2016), No 3–4, 69–74.

[3] Kuduk G., Symotyuk M. M., Problem with integral conditions for linear partial differential equations of second order, *Proceedings of International Conference. Boundary Value Problems Functional Equations Applications (Rzeszow, Poland, April 20–23, 2016)*, 3-rd Meeting, Faculty of Mathematics and Natural Sciences University of Rzeszow, 40 pp.

[4] Kalenyuk P. I, Nytrebych Z. M, Kuduk G, Symotyuk M. M., Integral problem for partial differential equation of higher order in an infinite strip, *Journal of Mathematical Sciences* **231** (2018), 495–506.

[5] Kuduk G., Problem with integral conditions for nonhomogeneous partial differential equations of fourth oreder, *Proceedings of XX International Scientific Mykhailo Kravchuk Conference (Kyiv, Ukraine, November 17–20, 2025)*, 77–78.

[6] Kuduk G., Problem with integral conditions for linear partial differential equations of third oreder in Sobolew space, *Proceedings of XX International Scientific Mykhailo Kravchuk ConferenceKyiv, Ukraine, November 17–20, 2025)*, 79–80.