On distinguishing Ising-type distributions

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The talk discusses the problem of distinguishing sequences of signs whose properties are close to those of stationary distributions in the Ising model.

Let $\{\xi_n; n \in \mathbb{Z}\}$ be independent standard Gaussian random variables. Define $\{\eta_n; n \in \mathbb{Z}\}$ as the unique stationary solution of the recurrence equation for $\alpha \in (-1, 1)$

$$\eta_{n+1} = a\eta_n + \xi_{n+1},\ n \in \mathbb{Z}.$$

Currently,

$$\eta_n = \sum_{k=0}^{\infty} \alpha^k \, \xi_{n-k}, \quad n \in \mathbb{Z}.$$

Let

$$x_n = \mathrm{sign}(\eta_n).$$

Lemma 1. $\{x_n; n \in \mathbb{Z}\}$ *is stationary, and*

$$Mx_0x_n = \frac{2}{\pi}\arcsin(\alpha^n).$$

Construct another sequence of signs $\{\tilde{x}_n; n \in \mathbb{Z}\}$ as a stationary Markov chain on {-1,1} with transition matrix

$$\begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix},\ p \in (0;1)$$

Lemma 2.

$$M\tilde{x}_0\tilde{x}_n = (p-q)^n,\ q = 1-p.$$

At the same time, $\{\tilde{x}_n; n \in \mathbb{Z}\}$ satisfies relations

$$P(\tilde{x}_n = 1 \mid \tilde{x}_{n-1} = \tilde{x}_{n+1} = 1) = \frac{p^2}{p^2+q^2},$$
$$P(\tilde{x}_n = 1 \mid \tilde{x}_{n-1} = \tilde{x}_{n+1} = -1) = \frac{q^2}{p^2+q^2},$$
$$P(\tilde{x}_n = 1 \mid \tilde{x}_{n-1} \neq \tilde{x}_{n+1}) = \frac{1}{2},$$

which are similar to the stationary distribution of the Ising model.

Theorem 1. *The mixing coefficient for each of the sequences* $\{\tilde{x}_n\}$ *and* $\{x_n\}$ *satisfies the condition*

$$\forall m \geq 1 \quad \sum_{n=1}^{\infty} \alpha_n \cdot n^m < +\infty.$$

Theorem 2.

$$\frac{1}{\sqrt{n}}\sum_{k=1}^{n} x_k \xrightarrow{d} N(0,\sigma^2),$$
$$\frac{1}{\sqrt{n}}\sum_{k=1}^{n} \tilde{x}_k \xrightarrow{d} N(0,\tilde{\sigma}^2),$$
$$n \to \infty,$$

where

$$\sigma^2 = 1 + 2\sum_{n=1}^{\infty} \arcsin\left(\frac{\alpha^{2n}}{1-\alpha^2}\right),$$
$$\tilde{\sigma}^2 = 1 + \frac{p-q}{q}.$$

The obtained result makes it possible to test the hypothesis on the distribution of a sequence of signs.

[1] Liggett T.M., Interacting Particle Systems, Springer-Verlag, New York, 1985, XIII+488 p.

[2] Utev S.A., The central limit theorem for ϕ-mixing arrays of random variables, *Theory of Probability and Its Applications* **35** (1990), no. 1, 131–139.

[3] Levin D.A., Peres Y., Wilmer E.L., Markov Chains and Mixing Times, 2nd ed., American Mathematical Society, Providence, 2017, 447 pp.