Polynomial vector fields tangent to a k-plane as a maximal Lie subalgebra

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Let K be an algebraically closed field of characteristic zero. Recall that a K-linear map $D\colon A \to A$ is called a K-derivation of a K-algebra A if it satisfies Leibniz's rule, i.e. $D(ab) = D(a)b + aD(b)$. The set of all K-derivations of a K-algebra A forms the Lie algebra $\mathrm{Der}_K A$.

The Lie algebra $W_n := \mathrm{Der}_K K[x_1, \ldots, x_n]$ of all polynomial K-derivations is of interest, since its elements can be interpreted as vector fields on the affine space K^n with polynomial coefficients. Although the structure of this Lie algebra has been studied extensively, little is known about its maximal subalgebras(see [1, 2]). The following result provides a family of maximal subalgebras of the Lie algebra W_n.

Theorem 1. *Let $I_k = (x_{k+1}, \ldots, x_n)$ be the ideal in the polynomial algebra $K[x_1, \ldots, x_n]$ generated by the last $n-k$ variables. Denote by L_k the subalgebra that preserves the ideal I_k, i.e., $L_k = \{D \in W_n \mid D(I_k) \subseteq I_k\}$. Then L_k is a maximal subalgebra of W_n for $k = 1, \ldots, n-1$.*

Remark 1. The Lie subalgebra L_k has a geometric interpretation. Namely, it consists of all polynomial vector fields tangent to the coordinate k-plane $x_{k+1} = 0, \ldots, x_n = 0$.

Remark 2. Conjugating the subalgebra L_k by polynomial automorphisms yields more maximal subalgebras in W_n.

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[2] Efimov D., Sydorov M., Sysak K., On maximality of some solvable and locally nilpotent subalgebras of the Lie algebra $W_n(K)$, *Researches in Mathematics* **31** (2023), no. 2, 17–25, arXiv:2310.05243.