Point symmetries of Kadomtsev–Petviashvili equation

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The famous Kadomtsev–Petviashvili equation

$$\left(u_t+\tfrac{3}{2}uu_x+\tfrac{1}{4}u_{xxx}\right)_x=\tfrac{3}{4}\sigma u_{yy},\quad \sigma=\pm1 \tag{1}$$

stands alongside the Korteweg–de Vries equation at the origins of modern integrability theory, which emerged in the 1970s. The study of these equations has led to groundbreaking developments such as the Hirota's bilinear method, the Miura's transform and Sato's hierarchy, while also revealing deep connections to representation theory and algebraic geometry [6].

The Lie symmetry analysis of (1) was carried out in the pioneering work [2]. Since then, the Kadomtsev–Petviashvili equation and its generalizations have become a very fruitful research field within the framework of Lie group analysis and group classification [3]. At the same time, it is surprising that the complete point-symmetry pseudogroup of (1) has never been presented in the literature. This gap is largely due to the limitations of traditional techniques and the inherently difficult computations required. In this work, we overcome these difficulties by applying both the direct method (see, e.g., [4]) and the megaideal-based version of the algebraic method (see, e.g., [1, 5]) to rigorously determine the complete point-symmetry pseudogroup of the Kadomtsev–Petviashvili equation (1).

Theorem 1. *The complete point-symmetry pseudogroup G of the Kadomtsev–Petviashvili equation* (1) *consists of the transformations*

$$\tilde t=T(t),\quad \tilde x=T_t^{1/3}x+\sigma\frac{2T_{tt}}{9T_t^{2/3}}y^2+\sigma\varepsilon\frac{2Y^0_t}{3T_t^{1/3}}y+X^0(t),\quad \tilde y=\varepsilon T_t^{2/3}y+Y^0(t),$$

$$\tilde u=\frac{1}{T_t^{2/3}}u+\frac{2T_{tt}}{9T_t^{5/3}}x+\frac{4\sigma}{9T_t^{1/3}}\left(\frac{T_{tt}}{3T_t^{4/3}}y^2+\varepsilon\frac{Y^0_t}{T_t}y\right)_t+\frac{2}{9T_t^2}\big(3T_tX^0_t-\sigma(Y^0_t)^2\big),$$

where T, X^0 and Y^0 are arbitrary smooth functions of t with $T_t\neq0$ and $\varepsilon=\pm1$.

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