International Conference of Young Mathematicians
The Institute of Mathematics of the National Academy of Sciences of Ukraine
June 3–5, 2026, Kyiv, Ukraine

Kantorovich-Rubinstein distance on Gaussian spaces

A. O. Bobrov[1]
[1]Kyiv Academic University, Kyiv, Ukraine
a.bobrov@kau.edu.ua

Consider a separable Banach space $(X, \|\cdot\|)$ equipped with a centered Gaussian measure μ on the Borel σ-algebra of X and assume $\mathrm{supp}(\mu) = X$.

Let $(H, |\cdot|_H)$ be the Cameron-Martin space of μ, i.e., the separable Hilbert space densely and continuously embedded in X such that

$$\int_X e^{i\cdot l(x)}\, d\mu(x) = \exp\left(-\frac{1}{2}|l|_H^2\right),\ l \in X^*. \tag{1}$$

Since H is continuously embedded in X, a functional $l \in X^*$ can be considered as a continuous linear functional on H. In (1) the expression $|l|_H$ denotes the norm of l as an element of H^*.

The space $\mathcal{M}(X)$ of Borel probability measures on X is endowed with the Kantorovich-Rubinstein distance

$$W_1(\nu_0, \nu_1) := \inf_{\gamma\in\Gamma(\nu_0,\nu_1)} \int_{X\times X} |x-y|_H\, d\gamma(x,y), \tag{2}$$

where $\Gamma(\nu_0, \nu_1)$ is the set of all Borel probability measures on $X \times X$ with marginals ν_0 and ν_1.

The following theorem was established in [1]:

Theorem 1. *Consider probability measures $\nu_0, \nu_1 \in \mathcal{M}(X)$ with $\nu_1 - \nu_0 \ll \mu$ and $\frac{d(\nu_1-\nu_0)}{d\mu} \in L^2(X, \mu)$. Then*

$$W_1(\nu_0, \nu_1) = \inf\left\{ \int_X |u(x)|_H\, d\mu(x) \,\middle|\, Iu = \frac{d(\nu_1-\nu_0)}{d\mu} \right\}, \tag{3}$$

where I denotes the *extended stochastic integral.*

Our main result is the Theorem 2.

Theorem 2. *Given the assumptions of Theorem 1, the infimum in* (3) *is attained for the case $X = \mathbb{R}^d$ $(d \geq 1)$. Equivalently, the* inf *operator can be replaced with* min.

[1] Riabov G. V., A representation for the Kantorovich-Rubinstein distance defined by the Cameron-Martin norm of a Gaussian measure on a Banach space, *Theory of Stochastic Processes*, **21**(37) (2016), no. 2, pp. 84-90.