The axisymmetrical poroelasticity problem for a radially multilayered hollow semi-infinite cylinder

Z. Yu. Zhuravlova[1]

[1]Odessa I.I. Mechnikov national university, Odessa, Ukraine

z.zhuravlova@onu.edu.ua

The multilayered hollow semi-infinite poroelastic cylinder ($a_0 < r < a_N, 0 < z < \infty, -\pi < \varphi < \pi$) is considered within the framework of Biot's model [1]. It consists of N radial layers $a_l < r < a_{l+1}, l = \overline{0, N-1}$. The ideal contact conditions are fulfilled between the layers

$$\begin{gathered} u_l(a_l, z) = u_{l+1}(a_l, z), \quad w_l(a_l, z) = w_{l+1}(a_l, z), p_l(a_l, z) = p_{l+1}(a_l, z), \\ \sigma_r^l(a_l, z) = \sigma_r^{l+1}(a_l, z), \quad \tau_{rz}^l(a_l, z) = \tau_{rz}^{l+1}(a_l, z), \quad k_l \frac{\partial p_l}{\partial r}(a_l, z) = k_{l+1} \frac{\partial p_{l+1}}{\partial r}(a_l, z), \end{gathered} \quad l = \overline{0, N-1} \tag{1}$$

where $u_l(r,z),\ w_l(r,z),\ p_l(r,z),\ \sigma_r^l(r,z),\ \tau_{rz}^l(r,z)$ are displacements, pore pressure, normal and tangential stress of l-th layer correspondingly, k_l is permeability coefficient.

The inner cylindrical surface $r = a_0$ is fixed and permeable

$$u_0(a_0, z) = 0, \quad w_0(a_0, z) = 0, \quad p_0(a_0, z) = 0, \tag{2}$$

The outer cylindrical surface $r = a_N$ is loaded by mechanical and fluid pressure loads

$$\sigma_r^{N-1}(a_N, z) = -L(z), \quad \tau_{rz}^{N-1}(a_N, z) = T(z), \quad p_{N-1}(a_N, z) = P(z), \tag{3}$$

where $L(z), \quad T(z), \quad P(z)$ are known functions.

The edge $z = 0$ is in ideal contact and impermeable conditions [2]

$$w(r, 0) = 0, \quad \tau_{rz}(r, 0) = 0, \quad \frac{\partial p}{\partial z}(r, 0) = 0. \tag{4}$$

The displacements, stress and pore pressure that satisfy conditions (1)–(4), equilibrium and storage equations [2] should be found.

The original problem is reduced to the one-dimensional boundary value problem with the help of semi-infinite sin-, cos- Fourier transforms applied regarding variable z. The derived problem is formulated as vector boundary value problem, and its solution is constructed with the help of matrix differential calculation apparatus [3] and recurrent correspondence [3]. The analytical expressions for displacements, stress and pore pressure are found for each layer. The numerical study was conducted for two- and three-layered cylinders with different external loadings.

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