ASYMPTOTIC PROPERTIES OF SOLUTIONS TO ONE CLASS OF NONLINEAR SECOND ORDER DIFFERENTIAL EQUATIONS

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Differential equation

$$y'' = \alpha_0 p(t) f(t, y, y'), \tag{1}$$

where $\alpha_0 \in \{-1; 1\}, p : [a, \omega[\rightarrow]0, +\infty[\; (-\infty < a < \omega \leq +\infty)$ is a continuous function, $f : [a, \omega[\times \Delta_{Y_0} \times \Delta_{Y_1} \rightarrow]0, +\infty[$ is continuously differentiable, $Y_i \in \{0, \pm\infty\}$, Δ_{Y_i} is either $[y_i^0, Y_i[$[1] or $]Y_i, y_i^0]$ is concidered. We also suppose the function f satisfy the conditions

$$\lim_{t \uparrow \omega} \frac{\pi_\omega(t) \cdot \frac{\partial f}{\partial t}(t, v_0, v_1)}{f(t, v_0, v_1)} = \gamma \text{ uniformly by } v_0 \in \Delta_{Y_0},\ v_1 \in \Delta_{Y_1}, \tag{2}$$

$$\lim_{\substack{y_k \to Y_k \\ y_k \in \Delta_{Y_k}}} \frac{v_k \cdot \frac{\partial f}{\partial v_k}(t, v_0, v_1)}{f(t, v_0, v_1)} = \sigma_k \quad \text{uniformly by } t \in [a, \omega[, \tag{3}$$

where $v_j \in \Delta_{Y_j}, j \neq k,\ k \in \{0, 1\}$.

By conditions (2), (3) the function f is in some sense close to regularly varying function [1] by every variable. Partial cases of (1) were considered for example in [2],[3]. For one class of regularly varying solutions to the equation (1) asymptotic representations and conditions of existence were found.

Definition 1. Solution y of the equation (1) is called $P_\omega(Y_0, Y_1, \lambda_0)$ if it is defined on $[t_0, \omega[\subset [a, \omega[$ and

$$\lim_{t \uparrow \omega} y^{(i)}(t) = Y_i \quad (i = 0, 1), \quad \lim_{t \uparrow \omega} \frac{(y'(t))^2}{y(t) y''(t)} = \lambda_0.$$

For different values of parameter λ_0 the class of such solutions contains regularly, slowly and rapidly varying as $t \uparrow \omega$ functions. $P_\omega(Y_0, Y_1, \lambda_0)$-solutions of the equation (1) are regularly varying functions as $t \uparrow \omega$ of index $\frac{\lambda_0}{\lambda_0 - 1}$ if $\lambda_0 \in R \setminus \{0, 1\}$.

For more general case as f depends only on y and y' asymptotic properties and necessary and sufficient conditions of existence of such solutions of equation (1) have been received in [2].

We need next subsidiary notations

$$\pi_\omega(t) = \begin{cases} t & \text{as } \omega = +\infty, \\ t - \omega & \text{as } \omega < +\infty, \end{cases} \qquad \Theta_i(z) = \varphi_i(z)|z|^{-\sigma_i}, \quad (i = 0, 1),$$

$$J_1(t) = \int_{A_\omega^1}^{t} \left(\alpha_0 p(\tau) \left| \pi_\omega(\tau) \right|^{\gamma + \sigma_0} \left| \frac{\lambda_0 - 1}{\lambda_0} \right|^{\sigma_0} \right) d\tau;$$

[1] As $Y_i = +\infty (Y_i = -\infty)$ assume $y_i^0 > 0$ $(y_i^0 < 0)$.

$$A_\omega^1 = \begin{cases} a, & \text{if } \int\limits_a^\omega p(\tau)\,|\pi_\omega(\tau)|^{\gamma+\sigma_0}\,d\tau = +\infty, \\ \omega, & \text{if } \int\limits_a^\omega p(\tau)\,|\pi_\omega(\tau)|^{\gamma+\sigma_0}\,d\tau < +\infty; \end{cases}$$

$$J_2(t) = |(1-\sigma_0-\sigma_1)|^{\frac{1}{1-\sigma_0-\sigma_1}}\, sign y_1^0 \int\limits_{B_\omega^2}^{t} |J_1(t)|^{\frac{1}{1-\sigma_0-\sigma_1}}\,d\tau;$$

$$B_\omega^2 = \begin{cases} b, & \text{if } \int\limits_b^\omega |J_1(t)|^{\frac{1}{1-\sigma_0-\sigma_1}}\,d\tau = +\infty, \\ \omega, & \text{if } \int\limits_b^\omega |J_1(t)|^{\frac{1}{1-\sigma_0-\sigma_1}}\,d\tau < +\infty; \end{cases}$$

Following theorem is obtained for the equation (1).

Theorem 1. *Let in the equation (1) $\sigma_1 \neq 1$. Then for the existence of $P_\omega(Y_0, Y_1, \lambda_0)$-solutions to the equation (1) in cases $\lambda_0 \in R \setminus \{0, 1\}$, it is necessary and if*

$$\lambda_0 \neq \sigma_1 - 1 \; or \; (\sigma_1 - 1)(\sigma_0 + \sigma_1 - 1) > 0,$$

then also sufficient

$$\pi_\omega(t) y_1^0 y_0^0 \lambda_0(\lambda_0 - 1) > 0, \;\; \pi_\omega(t)\alpha_0 y_1^0 \lambda_0(\lambda_0 - 1) > 0, \;\; as \;\; t \in [a, \omega[,$$

$$\lim_{t\uparrow\omega} y_0^0\,|\pi_\omega(t)|^{\frac{\lambda_0}{\lambda_0-1}} = Y_0, \;\; \lim_{t\uparrow\omega} y_1^0\,|\pi_\omega(t)|^{\frac{1}{\lambda_0-1}} = Y_1$$

$$\lim_{t\uparrow\omega} \frac{\pi_\omega(t) J_2'(t)}{J_2(t)} = \frac{\lambda_0}{\lambda_0 - 1}, \;\; \lim_{t\uparrow\omega} \frac{\pi_\omega(t) J_1'(t)}{J_1(t)} = \frac{1 - \sigma_0 - \sigma_1}{\lambda_0 - 1}.$$

Moreover, for each such solution, the following asymptotic representations hold as $t \uparrow \omega$

$$\frac{\pi_\omega(t) y'(t)}{y(t)} = \frac{\lambda_0}{\lambda_0 - 1}[1 + o(1)], \quad \frac{\pi_\omega(t) y''(t)}{y'(t)} = \frac{1}{\lambda_0 - 1}[1 + o(1)].$$

1. Bingham N.H., Goldie C.M., Teugels J.L. Regular Variation. Cambridge: Cambridge University Press, 1987, 512 p.
2. Bilozerova M. O., Herzhanovs'ka H. A. Asymptotic Behavior of the Solutions of Essentially Nonlinear Nonautonomous Second-Order Differential Equations Close to Linear Functions, Journal of Mathematical Sciences, 2023, 274 (1), P. 1–12.
3. Gerzhanovskaya G. A. Properties of slowly varying solutions to essentially nonlinear second order differential equations, Bukovinian mathematical journal, 2017, 5 (3-4), P. 39–46.