Poisson process of order k with time change

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Let $N^k(t)$ denote the process, with the associated probability mass function (PMF) given by:

$$p_n^{N^k}(t) = \mathbb{P}(N^k(t) = n) = \sum_{X\in\Omega(k,n)} e^{-k\lambda t} \frac{(\lambda t)^{\zeta_k}}{\prod_k !}$$

where $\Omega(k,n) = \{X = (x_1, x_2, \ldots, x_k) | x_1 + 2x_2 + \cdots + kx_k = n\}$, $x_1, x_2, \ldots, x_k$ are non-negative integers, $\zeta_k = x_1 + x_2 + \cdots + x_k$, $\prod_k ! = x_1!x_2! \cdots x_k!$. This process is called Poisson process of order k. It was introduced in the paper by Kostadinova and Minkova [1] and has been actively studied in recent literature.

In this work, we investigate time-changed Poisson processes of order k where the time change is governed by the following processes:

- the Compound Poisson-Gamma process,
- the Inverse Compound Poisson-Exponential process,
- the Bessel subordinator.

Our main focus is on derivation and analysis of the PMFs of the resulting time-changed processes. These PMFs are expressed in terms of special functions, including:

- the Wright function,
- the generalized Wright function,
- the two-parameter and three-parameter generalized Mittag-Leffler functions,
- the modified Bessel function of the first kind.

The usage of these functions allows for compact analytical representations of the distributions. The difference-differential equations for distributions of time-changed processes are also derived.

Detailed results and applications will be discussed during the presentation. The talk is based on joint work with L. Sakhno.

1. Kostadinova K., Minkova L. On the Poisson process of order k. Pliska Stud. Math. Bulgar., 2012, 22, 117–128.
2. Buchak K., Sakhno L. Compositions of Poisson and Gamma processes. Modern Stoch. Theory Appl., 2017, 4, 2, 161–188.