DYNAMICS OF CONFLICT INTERACTION IN TERMS OF MINIMAL VALUES

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Let two opponents, A and B, represented as players, be assigned independent discrete random distributions at time $t = 0$ over the space $\Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$, where $n \geq 2$ represents the set of visited positions. These distributions are defined by vectors $\mathbf{p} = (p_1, \ldots, p_n)$ and $\mathbf{r} = (r_1, \ldots, r_n)$, where p_i and r_i are the probabilities of player A and player B being in position ω_i, respectively.

We assume that the vectors $\mathbf{p}$ and $\mathbf{r}$ are stochastic, meaning that:

$$0 \leq p_i, r_i \leq 1, \quad \sum_{i=1}^{n} p_i = \sum_{i=1}^{n} r_i = 1.$$

Furthermore, the vectors $\mathbf{p}$ and $\mathbf{r}$ are assumed to be different and non-orthogonal, implying that their scalar product satisfies $0 < (\mathbf{p}, \mathbf{r}) < 1$.

At subsequent discrete times $t = 1, 2, \ldots$, players A and B interact with each other, resulting in changes to their distributions. The dynamics of these changes are governed by the following recursive system of equations:

$$p_i^{t+1} = \alpha \frac{1}{z_p^t} p_{\min}^t \left(1 - r_{\min}^t\right), \quad r_i^{t+1} = \alpha \frac{1}{z_r^t} r_{\min}^t (1 - p_{\min}^t),$$

where $p_{\min}^t$ and $r_{\min}^t$ represent the minimum values of the coordinates of the vectors $\mathbf{p}^t$ and $\mathbf{r}^t$, respectively, and z_p^t, z_r^t are normalizing denominators ensuring the stochastic nature of the vectors.

This system is studied with respect to the parameter α and the dimensionality of the vectors $\mathbf{p}^t$, $\mathbf{r}^t \in \mathbb{R}_+^n$. Similar dynamic system models have been constructed and studied in the works

Similar models of dynamic systems of this type have been constructed and investigated in the works [1]–[12]. The study presents a detailed analysis of conflict dynamics, focusing on the behavior of the system in terms of minimal players. The existence of stationary states and cyclic orbits in the dynamics was analyzed, as well as the stability of the system under various conditions. In particular, this work provides a detailed analysis of the conflict dynamics when the number of positions is reduced to two, i.e., for $n = 2$. This special case is of significant interest as it simplifies the system and provides more tractable insights into the fundamental behaviors of the dynamics. In this case, the distribution vectors $\mathbf{p}^t$ and $\mathbf{r}^t$ are reduced to two components, allowing for more direct analysis of the interactions and their outcomes. The behavior of the system is analyzed with respect to the interaction strength α, and the properties of stationary states and cyclic behavior are investigated in the context of this two-position system. A detailed analysis of this dynamic system was published in the work [13].

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