Automorphism group of some Leibniz algebras

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Let L be an algebra over a field F with the binary operations $+$ and $[,]$. Then L is called a (*left*) *Leibniz algebra* if it satisfies the left Leibniz identity:

$$[[a,b],c]=[a,[b,c]]-[b,[a,c]]$$

for all elements $a,b,c\in L$ [1,2].

A linear transformation f of L is called an *endomorphism* of L, if

$$f([a,b])=[f(a),f(b)]$$

for all elements $a,b\in L$. A bijective endomorphism of L is called an *automorphism* of L. We note that the set $\mathrm{Aut}_{[,]}(L)$ of all automorphisms of L is a group by a multiplication.

Consider the following type of 3-dimensional non-nilpotent Leibniz algebras:

$$L=Fa_1\oplus Fa_2\oplus Fa_3,\ \text{where } [a_1,a_1]=[a_1,a_3]=a_3,$$
$$[a_1,a_2]=[a_2,a_1]=[a_2,a_2]=[a_2,a_3]=[a_3,a_1]=[a_3,a_2]=[a_3,a_3]=0.$$

Thus, $\mathrm{Leib}(L)=[L,L]=Fa_3$, $\zeta^{\mathrm{left}}(L)=Fa_2\oplus Fa_3$, $\zeta^{\mathrm{right}}(L)=\zeta(L)=Fa_2$.

Theorem 1. *Let G be the automorphism group of a Leibniz algebra L. Then G is isomorphic to a subgroup of $\mathrm{GL}_3(F)$ consisting of matrices of the following form:*

$$\begin{pmatrix}\alpha_1 & 0 & 0\\ \alpha_2 & \beta_2 & \gamma_2\\ \alpha_3 & 0 & \alpha_1+\alpha_3\end{pmatrix},$$

where $\alpha_1,\alpha_2,\alpha_3,\beta_2,\gamma_2\in F$, $\alpha_1\neq 0$, $\beta_2\neq 0$, $\alpha_1+\alpha_3\neq 0$.

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