Rotation number of the Gaussian random field on the plane

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Definition 1. The rotation number $\gamma(\xi, L)$ of the continuous vector field $\xi : \mathbb{R}^2 \to \mathbb{R}^2$ along the continuous curve $L : [0,1] \to \mathbb{R}^2$ is the number $\frac{1}{2\pi}(\phi(1) - \phi(0))$, where ϕ is the continuous branch of the angular function of ξ.

The main object of study is the Gaussian stationary random field $\xi : R^2 \supseteq U \to R^2$ with covariance function

$$\mathbb{E}\xi_i(u)\xi_i(v) = e^{-||u-v||^2}.$$

We investigate the rotation number of ξ on the boundary of compact set $B \subset \mathbb{R}^2$ (further denoted as $\gamma(\xi, B)$). We assume that the boundary of B is a closed curve with continuous parametrization $\psi : [0,1] \mapsto \mathbb{R}^2$. The rotation number can be calculated as follows [1]:

$$\gamma(\xi, B) = \sum_{u \in B : \xi(u) = 0} sign(\det(\xi'(u)))$$

where

$$\det \xi'(u) = \frac{\partial \xi_x}{\partial x}(u)\frac{\partial \xi_y}{\partial y}(u) - \frac{\partial \xi_y}{\partial x}(u)\frac{\partial \xi_x}{\partial y}(u)$$

We prove the following lemmata.

Lemma 1.

$$P(\{\exists u \in B : \xi(u) = 0, \det \xi'(u) = 0\}) = 0$$

Lemma 2.

$$\mathbb{E}\gamma(\xi, B) = \int_B \int_{\mathbb{R}^4} \det(y) p_{\xi(u),\xi'(u)}(0, y) dy du$$

where $p_{\xi(u),\xi'(u)}(x, y)$ *is the joint density of* ξ, ξ' *in the point* $u \in \mathbb{R}^2$.

Lemma 3.

$$\gamma(\xi, B) = \gamma(\xi, B + s)$$

in distribution, for all $s \in \mathbb{R}^2$.

1. Krasnoselskii M., Perov A., Povolockii A., Zabreiko P. Plane vector fields. — Academic Press, 1966, 243 p.