Problem with integral conditions for nonhomogeneous system of partial differential equations third order

G. Kuduk

Faculty of Mathematics and Natural Sciences, University of Rzeszow, Graduate of University

gkuduk@onet.eu

Let $H(\mathbb{R}_+ \times \mathbb{R}^n)$ be a class of certain function, $K_{L,M}$ be a class of quasi-polynomials of the form

$$f(t,x) = \sum_{i=1}^{n} Q_i(t,x) e^{\alpha_i x + \beta_j t}, \tag{1}$$

where $Q_{ij}(t,x)$ are given polynomials, $M \subseteq \mathbb{C}$, $\alpha_i \in M$, $\alpha_k \neq \alpha_l$, for $k \neq l$, $\beta_j \in M$, $\beta_k \neq \beta_l$, for $k \neq l$. Each quasi-polynomial (1) defines a differential operator $f\left(\frac{\partial}{\partial\nu}, \frac{\partial}{\partial\lambda}\right)$ of finite order on the class of certain function, in the form

$\displaystyle\sum_{i=1}^{n} Q_{ij}\left(\frac{\partial}{\partial\nu}, \frac{\partial}{\partial\lambda}\right) \exp\left[\alpha_i \frac{\partial}{\partial\lambda} + \beta_j \frac{\partial}{\partial\nu}\right]\Bigg|_{\lambda=\nu=0}$. Let be $T_{kjp}(t,\lambda) =$
$= \widetilde{l}\left(\frac{d}{dt}, \lambda\right) W(t,\lambda)$, $j = 1,...,n$, $p = 1,...,n$, $k = 1,2$, satisfies of system of equations $\sum_{j=1}^{n} l_{ij}\left(\frac{d}{dt}, \lambda\right) T_j(t,\lambda) = 0$, $i = 1,...,n$, where $\left(\frac{d}{dt}, \lambda\right) = \delta_{ij}\frac{d^3}{dt^3} - a_{ij}\frac{d^2}{dt^2} - b_{ij}\frac{d}{dt}$, $\delta_{ij} - c_{i,j}$ - symbol Kroneckera. Let $L(\lambda,\nu) = \|L(\nu,\lambda)\|_{ij=1,...,n}$, $\psi(\nu,\lambda) = \det L(\nu,\lambda)$, $\widetilde{l}\left(\frac{d}{dt}, \lambda\right)$ - algebraic component element $\left(\frac{d}{dt}, \lambda\right)$ is matrix $L(\lambda,\nu)$. $W(t,\lambda)$ is a solution of the problem $\psi\left(\frac{d}{dt}, \lambda\right) W(t,\lambda) = 0$, satisfies conditions $W^j(0,\lambda) = \delta_{j,2n-1}$, $j = 2n-1$, let be $\eta(\lambda)$ ba a certain function.

Denote be

$$P = \{\lambda \in \mathbb{C} : \eta(\lambda) = 0\}. \tag{2}$$

In the strip $\Omega = \{(t,x) \in \mathbb{R}^{n+1} : t \in (0,T), x \in \mathbb{R}^n\}$ we consider problem

$$\frac{\partial^3 U_i}{\partial t^3} + \sum_{j=1}^{n} \left\{ a_{ij}\left(\frac{\partial}{\partial x}\right) \frac{\partial^2 U_i}{\partial t^2} + b_{ij}\left(\frac{\partial}{\partial x}\right) \frac{\partial}{\partial t} + c_{ij}\left(\frac{\partial}{\partial x}\right) \right\} U_j(t,x) = f_i(t,x), \tag{3}$$

$$\int_{T_1}^{T_2} t^k U_i(t,x)dt + \int_{T_3}^{T_4} t^k U_i(t,x)dt = 0, \quad k = \{0,1,2\}, \quad i = \{1,...,n\}, \tag{4}$$

where $a_{ij}\left(\frac{\partial}{\partial x}\right)$, $b_{ij}\left(\frac{\partial}{\partial x}\right)$, are differential expressions with entirel functions $a_{ij}(\lambda) \neq 0, b_{ij}(\lambda) \neq 0$

Theorem 1. *Let $f_i(t,x) \in K_{L,M}$, $i = \{1,...,n\}$, $k = \{1,2\}$, then the class $K_{M\setminus P}$ exist and unique solution of the problem (3),(4), can be represented in the form*

$$U_j(t,x) = \sum_{k=0}^{2} \sum_{p=1}^{n} f_{kp}\left(\frac{\partial}{\partial\lambda}, \frac{\partial}{\partial\nu}\right) \left\{ \frac{1}{\eta(\lambda)} T_{kjp}(t,\lambda) \exp[\lambda x] \right\}\Bigg|_{\lambda=\nu=0}.$$

where P is set (2).

Solution of the problem (3)-(4) according to the differential-symbol method [1,2], exists and uniqess in the class of quasi-polynomials. This result continues the research of work [3–7].

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