ON A MAXIMAL SUBALGEBRA OF THE LIE ALGEBRA $W_2(\mathbb{K})$

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Let $\mathbb{K}$ be an algebraically closed field of characteristic zero, and let $P_n = \mathbb{K}[x_1, \ldots . x_n]$ be the polynomial ring in n variables. Let us recall that a $\mathbb{K}$-linear map $D : P_n \to P_n$ is called a $\mathbb{K}$-derivation on P_n if $D(fg) = D(f)g + fD(g)$ for any $f, g \in P_n$. The vector space $W_n(\mathbb{K})$ of all derivations on P_n is a Lie algebra over the field $\mathbb{K}$ with the Lie bracket $[D_1, D_2] = D_1D_2 - D_2D_1$ for any $D_1, D_2 \in W_n(\mathbb{K})$ and from the geometrical point of view $W_n(\mathbb{K})$ consists of all polynomial vector fields on $\mathbb{K}$, which makes this issue particularly interesting. This Lie algebra was studied by many authors from different points of view (see, e.g., [1],[2],[3]). In particular, in [1] and [3] some maximal subalgebras from $W_n(\mathbb{K})$ were pointed out. In general, there is no description of maximal subalgebras of the Lie algebra $W_n(\mathbb{K})$, so concrete examples of such subalgebras are of interest.

A maximal subalgebra of $W_n(\mathbb{K})$ is a proper subalgebra $M \subset W_n(\mathbb{K})$, i.e. $M \neq W_n(\mathbb{K})$, such that $W_n(\mathbb{K})$ itself is the only Lie subalgebra properly containing M. Some cases with additional restrictions (abelian Lie subalgebras or subalgebras with zero/constant divergence) are also of great interest.

The next theorem describes the maximal subalgebra of the Lie algebra $W_2(\mathbb{K})$ and states some related structural properties. It is important, because $W_2(\mathbb{K})$ represents algebraic vector fields on the affine plane, which are central to algebraic geometry and automorphism groups. And also the simplest case where non-trivial symplectic and Hamiltonian structures appear.

Theorem 1. *Let $M_1(\mathbb{K})$ be a subalgebra of the Lie algebra $W_2(\mathbb{K})$ of the form*

$$M_1(\mathbb{K}) = \left\{ f(x_1)\frac{\partial}{\partial x_1} + g(x_1, x_2)\frac{\partial}{\partial x_2} \mid f, g \in \mathbb{K}[x_1, x_2] \right\}.$$

Then $M_1(\mathbb{K})$ is a maximal subalgebra of $W_2(\mathbb{K})$, the set $G = \left\{ g(x_1, x_2)\frac{\partial}{\partial x_2} \mid g \in \mathbb{K}[x_1, x_2] \right\}$ is an ideal of the Lie algebra $M_1(\mathbb{K})$ and $G \simeq W_1(\mathbb{K}) \otimes P_1$.

The factor-algebra $M_1(\mathbb{K})/G$ is isomorphic to the Lie algebra $W_1(\mathbb{K})$.

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