A law of the iterated logarithm for small counts in Karlin's occupancy scheme

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In the Karlin infinite occupancy scheme, balls are thrown independently into an infinite array of boxes 1, 2, ..., with probability p_k of hitting the box k. For $j, n \in \mathbb{N}$, denote by $\mathcal{K}_j^*(n)$ and $\mathcal{K}_j(n)$ the number of boxes containing exactly and at least j balls, respectively, provided that n balls have been thrown. We call *small counts* the variables $\mathcal{K}_j^*(n)$, with j fixed.

In [1], a law of the iterated logarithm (LIL) for $\mathcal{K}_j(n)$ as $n \to \infty$ was proved. Its proof exploits a Poissonization technique and is based on a new LIL for infinite sums of independent indicators $\sum_{k\geq 1} \mathbb{1}_{A_k(t)}$ as $t \to \infty$, where the family of events $(A_k(t))_{t\geq 0}$ is nondecreasing in t.

The crucial difference between $\mathcal{K}_j(n)$ and $\mathcal{K}_j^*(n)$ is that the former process in nondecreasing in n, while the latter is not. Therefore, application of the aforementioned LIL to *small counts* is impossible.

I will discuss how we lifted the monotonicity assumption in [2] and obtained the extended LIL for infinite sums of independent indicators $\sum_{k\geq 1} \mathbb{1}_{A_k(t)}$ as $t \to \infty$, where the family of events $(A_k(t))_{t\geq 0}$ is ***not necessarily monotone*** in t. As a corollary, we prove LIL for the small counts as the number of balls thrown becomes large.

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