# On one family of self-similar sets with nontrivial features

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Depending on parameter $l \in \mathbb{N}$, we consider a self-similar set

$$K_l = \left\{ \sum_{i=1}^{\infty} \frac{\varepsilon_i}{(2l+2)^i} : (\varepsilon_i) \in \{0, 2, 4, \dots, 2l, 2l+1, 2l+3, \dots, 4l+1\}^{\mathbb{N}} \right\}$$

generated by a homogeneous iterated function system (IFS) $\Psi_l = \{\psi_i(x)\}_{i=1}^{2l+2}$ defined by the following similarities:

$$\begin{cases} \psi_i(x) = \frac{x+(2i-2)}{2l+2} & \text{if} \quad 1 \le i \le l+1, \\ \psi_i(x) = \frac{x+(2i-3)}{2l+2} & \text{if} \quad l+2 \le i \le 2l+2. \end{cases}$$

In this talk, we show that $K_l$ simultaneously exhibits properties of both intervals and Cantor-like sets. Moreover, $K_l$ is a perfect set that coincides with the closure of its interior. In particular, the set $K_l$ possesses the following properties:

- The interval $\left[0, \frac{4l+1}{2l+1}\right]$ serves as the convex hull of $K_l$.
- $K_l$ is symmetric with respect to its midpoint $\frac{4l+1}{4l+2}$.
- The interval $\left[\frac{2l}{2l+1}, 1\right]$ is entirely contained within the set $K_l$.
- $K_l \setminus \left(\frac{2l}{2l+1}, 1\right)$ is the union of pairwise disjoint affine copies of $K_l$. Moreover, this union contains $2l$ isometric copies of $K_l/(2l+2)^i$ for each $i \in \mathbb{N}$.

Based on these properties, our main result is summarized in the following theorem.

**Theorem 1.** *The interior of the set $K_l$ is a countable union of open intervals whose total length equals 1, while its boundary is a Cantor-like set of zero Lebesgue measure and Hausdorff dimension* $\log(2l+1)/\log(2l+2)$.

It is important to note that for each $l \in \mathbb{N}$, the set $K_l$ is generated by an iterated function system satisfying the Open Set Condition. This demonstrates that even in seemingly simple case, the resulting attractor can exhibit highly intricate structures. Related families of sets with similar properties have also been studied in [1].

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1. D. Karvatskyi, M. Pratsiovytyi, and O. Makarchuk, Fractal analysis of Guthrie-Nymann's set and its generalisation, Ukrainian Mathematical Journal (under review), 2025. [Available at arXiv:2405.16576].