An analytical approximation of the solutions of neutral stochastic differential equations

D. D. Djordjević, M. Jovanović, M. Milošević
Faculty of Sciences and Mathematics, University of Niš, Niš, Serbia
dusan.djordjevic@pmf.edu.rs, miljana.jovanovic@pmf.edu.rs, marija.milosevic@pmf.edu.rs

Many natural and social phenomena, such as ones from physics, medicine, chemistry, economy, and biology, for example, may be represented by neutral stochastic differential equations (NSDEs). Therefore, the studies of NSDEs are of great importance. However, just like most of equations in mathematics, these equations are also non-solvable in most cases. Those NSDEs are of interest. In [1], the authors examined NSDEs with constant delay

$$x(t) - N\big(t, x(t-\mu)\big) = \xi(0) - N\big(t_0, \xi(-\mu)\big) + \int_{t_0}^{t} F\big(s, x(s), x(s-\mu)\big)ds + \int_{t_0}^{t} G\big(s, x(s), x(s-\mu)\big)dw_s, \quad t \in [t_0, T],$$

and presented an analytical approximate solution to these equations under certain conditions. The conditions are not the most common ones (Lipschitz and/or linear growth conditions), since the desire is to broaden the class of equations for which the method can be used. The above-mentioned conditions are polynomial conditions, to go along with other auxiliary assumptions. The approximation is based on the Taylor expansion with respect to the delayed argument of N, F and G. The existence and uniqueness of thusly formed NSDE is proven and the closeness between solutions of the equations is estimated in the almost sure and L^p sense. Higher the derivative orders in Taylor expansions are, closer are the solutions, as expected. The results derived from different techniques, as well as various probability, integral, martingale-related and some known elementary inequalities. These results will be presented here, as well as two examples which show that the class of observed equations is nonempty.

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1. Djordjević D. D., Jovanović M., Milošević M. A Taylor approximation of the solutions of neutral stochastic differential equations with constant delay under polynomial conditions. Journal of the Korean Mathematical Society, in press.