Least squares method in the theory of nonlinear periodic boundary value problems with concentrated delay in critical cases

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Among numerous studies of functional-differential equations, research on periodic boundary value problems for differential equations with concentrated delay holds a special place [1]. This is primarily due to the wide application of periodic boundary value problems for differential equations with concentrated delay in physics, chemistry, economics, biology and mechanics [1,2,3].

The construction of solutions for nonlinear boundary value problems using the method of simple iterations and the least squares method is significantly complicated by the computation of derivatives of the nonlinearities [1,4,5,6,7,8]. Given this, simplifying the computation of nonlinear derivatives and the potential to find solutions for nonlinear boundary value problems, including periodic boundary value problems, in the form of elementary functions can be achieved using the Adomian decomposition method. Additionally, the use of the Adomian decomposition method significantly simplifies the proof of convergence of iterative schemes for constructing solutions to nonlinear boundary value problems [5,9,10,11,12,13].

By applying the hybrid technique based on lest square method and Adomian decomposition method, we have derived the necessary and sufficient conditions for the existence of solutions to the weakly nonlinear periodic boundary value problem for a system of differential equations with concentrated delay in the critical and noncritical cases [1,14,15]. As examples of application of the constructed iterative scheme, we obtain approximations to the solutions of a periodic boundary value problem for the mathematical model of non-isothermal chemical reactions [2,3,16]. To check the accuracy of the proposed approximations, we evaluate discrepancies in the original equation with concentrated delay [17,18,19].

The scientific research of co-author V. Chuiko was partially supported by:

- the 2024 NAS of Ukraine Prize for Young Scientists and Students;
- the 2024 ICMU Prize for Student Research in Mathematical Sciences;
- the 2024 Hansel and Gudi Tsap Prize.

In addition, V. Chuiko was partially supported by a grant from Department of Targeted Training of Taras Shevchenko National University of Kyiv under the National Academy of Sciences of Ukraine for 2024-2025.

Dedicated to the memory of Academician of the National Academy of Sciences of Ukraine Oleksandr Andriyovych Boichuk (30.06.1950–20.06.2024).

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