Asymptotic Behaviour of Slowly Varying Solutions of Second Order Differential Equations with the Product of Regularly and Rapidly Varying Nonlinariaties of an Unknown Function and Its First-order Derivative

O. O. Chepok

South Ukrainian National Pedagogical University named after K. D. Ushinsky, Odesa, Ukraine

olachepok@ukr.net

We consider the following differential equation

$$y'' = \alpha_0 p(t)\varphi_0(y')\varphi_1(y), \tag{1}$$

where $\alpha_0 \in \{-1, 1\}$, functions $p\colon [a, \omega[\to]0, +\infty[$, $(-\infty < a < \omega \le +\infty)$ and $\varphi_i : \Delta_{Y_i} \to]0, +\infty[$ $(i \in \{0, 1\})$ are continuous, $Y_i \in \{0, \pm\infty\}$, Δ_{Y_i} is the some one-sided neighborhood of Y_i.

We also suppose that φ_1 is a regularly varying as $y \to Y_1$ function of index σ_1 (see, [4, p. 10]), function φ_0 is twice continuously differentiable on Δ_{Y_0} and satisfies the following conditions

$$\varphi_0'(y') \neq 0 \ \text{ as } \ y' \in \Delta_{Y_0}, \quad \lim_{\substack{y' \to Y_0 \\ y' \in \Delta_{Y_0}}} \varphi_0(y') \in \{0, +\infty\}, \quad \lim_{\substack{y' \to Y_0 \\ y' \in \Delta_{Y_0}}} \frac{\varphi_0(y')\varphi_0''(y')}{(\varphi_0'(y'))^2} = 1. \tag{2}$$

It follows from the above conditions (2) that the function φ_0 and its derivative of the first order are rapidly varying functions as the argument tends to Y_0 [1]. So we have that the equation (1) is the second order differential equation that contains the product of a regularly varying function of unknown function and a rapidly varying function of its first derivative in its right-hand side. For the equation (1), the following class of solutions is considered

The solution y of the equation (1), that is defined on the interval $[t_0, \omega[\subset [a, \omega[$, is called $P_\omega(Y_0, Y_1, \lambda_0)$-solution $(-\infty \le \lambda_0 \le +\infty)$, if the following conditions take place

$$y^{(i)} : [t_0, \omega[\longrightarrow \Delta_{Y_i}, \quad \lim_{t\uparrow\omega} y^{(i)}(t) = Y_i \ \ (i = 0, 1), \quad \lim_{t\uparrow\omega} \frac{(y'(t))^2}{y''(t)y(t)} = \lambda_0.$$

The results for $P_\omega(Y_0, Y_1, \lambda_0)$–solutions of the equation (1) in cases $\lambda_0 \in R \setminus \{0; 1\}$ and $\lambda_0 = \pm\infty$ were obtained in the works [2,3].

In the work we have established the necessary and sufficient conditions for the existence of $P_\omega(Y_0, Y_1, \lambda_0)$–solutions of the equation (1) in case $\lambda_0 = 0$. Also we have found asymptotic representations of such solutions and its first order derivatives as $t \uparrow \omega$.

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