Haptotaxis-Driven Cancer Invasion Dynamics: A Stabilized Finite Element Approach

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This study explores the computational modeling of haptotaxis-driven tumor invasion, emphasizing nonlinear, time-dependent interactions governed by cross-diffusion mechanisms. The presence of dominant advection components in such models often leads to numerical instabilities, resulting in spurious oscillations and nonphysical negative densities in conventional finite element formulations. To address these challenges, we introduce a stabilized finite element framework that leverages the streamline-upwind/Petrov–Galerkin (SUPG) [1, 2] method for spatial discretization. Additionally, a discontinuity-capturing operator [3, 4, 5] is incorporated to improve solution accuracy, particularly in regions with sharp gradients. Temporal discretization is handled using the Crank–Nicolson scheme to ensure stability.

The computational implementation is carried out within the FEniCS open-source platform [6], enabling the simulation of multiple haptotaxis models. Comparative analyses show that while classical finite element and SUPG schemes exhibit numerical instabilities under advection-dominated conditions—often leading to negative species densities—the proposed stabilization strategy, enhanced with discontinuity-capturing, successfully preserves physical consistency.

Our results demonstrate the effectiveness of the proposed approach in simulating tumor invasion dynamics in two-dimensional domains, providing insights into cancer progression, therapy-driven tumor responses, and treatment optimization.

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