On codes of length 24 and the group algebra $F_3(C_3 \times D_8)$

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The previous results presented in [1], amoung others, were related to binary codes, namely the extended binary Golay codes. The construction of these codes by group algebras of different groups of order 24 was considered.

We consider codes of length 24 and their relation with the elements of the group algebra $F_3(C_3 \times D_8)$. Let $\mathbb{F}_3$ be a field with three elements, $G = \{g_1, g_2, \ldots, g_{24}\}$ be a finite group of order 24 and $u = \alpha_{g_1} g_1 + \alpha_{g_2} g_2 + \ldots + \alpha_{g_{24}} g_{24}$ be an element of the group algebra $\mathbb{F}_3 G$ ($\alpha_i \in \mathbb{F}_3$). Let $u \to \sigma(u)$ be a map, where $\sigma(u) \in M(n, \mathbb{F}_3)$ is the following matrix

$$\sigma(u) = \begin{pmatrix} \alpha_{g_1^{-1} g_1} & \alpha_{g_1^{-1} g_2} & \cdots & \alpha_{g_1^{-1} g_n} \\ \alpha_{g_2^{-1} g_1} & \alpha_{g_2^{-1} g_2} & \cdots & \alpha_{g_2^{-1} g_n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{g_n^{-1} g_1} & \alpha_{g_n^{-1} g_2} & \cdots & \alpha_{g_n^{-1} g_n} \end{pmatrix}.$$

We define for a given element $u \in \mathbb{F}_3 G$ a code: $C(u)$ is a subspace of space $\mathbb{F}_3^n$ generated by the rows of the matrix $\sigma(u)$.

Let $G = \langle x, y, z | x^3 = 1, y^4 = 1, z^2 = 1, xy = yx, xz = zx, yzyz = 1 \rangle$ be the $C_3 \times D_8$ group and $u \in \mathbb{F}_3(C_3 \times D_8)$. As a result of calculations we obtain the number of elements $u \in \mathbb{F}_3 G$ that $C(u)$ is a code of length 24 where the dimension of the subspace of codewords is 12 and the condition $\sigma(u) = \sigma(u)^T$ is met.

1. Maria Yu. Bortos, Alexander A. Tylyshchak, Myroslava V. Khymynets. Extended binary Golay codes by a group algebra. Algebra Discrete Math., 2024, 38(1) 23−33. http://dx.doi.org/10.12958/adm2241.