ON ISOMORPHISMS OF ALGEBRAS OF ENTIRE SYMMETRIC FUNCTIONS ON BANACH SPACES

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The talk will provide an overview of the results of the article [1]. We show that Fréchet algebras of symmetric entire functions of bounded type on complex Banach spaces are isomorphic if semigroups of symmetries on underlying Banach spaces satisfy some natural conditions.

Definition 1. Let A, B be arbitrary nonempty sets and S be an arbitrary fixed set of mappings $s\colon A \to A$. A mapping $f\colon A \to B$ is called S-symmetric if $f(s(a)) = f(a)$ for every $a \in A$ and $s \in S$.

Definition 2. Let X be a complex Banach space and $H_b(X)$ be the Fréchet algebra of all entire functions $f\colon X \to \mathbb{C}$ which are bounded on bounded sets endowed with the topology of uniform convergence on bounded sets. Let $\|f\|_r = \sup_{\|x\| \leqslant r} |f(x)|$ for $f \in H_b(X)$ and $r > 0$. The topology of $H_b(X)$ can be generated by an arbitrary set of norms $\{\|\cdot\|_r : r \in \Gamma\}$, where Γ is any unbounded subset of $(0, +\infty)$.

Let X be a complex Banach space and S be a set of operators on X. Let $H_{b,S}(X)$ be the subalgebra of all S-symmetric elements of $H_b(X)$.

Theorem 1. *Let X and Y be complex Banach spaces. Let S_1 and S_2 be semigroups of operators on X and Y resp. Let $\iota\colon X \to Y$ be an isomorphism such that*

1) *for every $x \in X$ and $s_1 \in S_1$, there exists $s_2 \in S_2$ such that $\iota(s_1(x)) = s_2(\iota(x))$;*

2) *for every $y \in Y$ and $s_2 \in S_2$, there exists $s_1 \in S_1$ such that $\iota^{-1}(s_2(y)) = s_1(\iota^{-1}(y))$.*

Then

a) $\|g \circ \iota\|_r \leqslant \|g\|_{r\|\iota\|}$ *for every $g \in H_{b,S_2}(Y)$ and $r > 0$;*

b) $g \circ \iota \in H_{b,S_1}(X)$ *for every $g \in H_{b,S_2}(Y)$;*

c) $f \circ \iota^{-1} \in H_{b,S_2}(Y)$ *for every $f \in H_{b,S_1}(X)$;*

d) *the mapping*
$$I\colon g \in H_{b,S_2}(Y) \mapsto g \circ \iota \in H_{b,S_1}(X)$$
is an isomorphism, i. e., I is a continuous linear multiplicative bijection.

We apply this result to Fréchet algebras of symmetric entire functions on Cartesian powers of complex Banach spaces of Lebesgue integrable in a power $p \in [1, +\infty)$ functions and of Lebesgue measurable essentially bounded functions on $[0, 1]$. Let X be one of the above-mentioned spaces. We show that the Fréchet algebra of all entire symmetric functions of bounded type on the Cartesian power X^n, where $n \in \mathbb{N}$, is isomorphic to the Fréchet algebra of all entire n-block symmetric functions of bounded type on X. In order to obtain this result we also prove some properties of the group of bijections of $[0, 1]$ that generates the group of symmetries for block symmetric functions.

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1. Vasylyshyn T., Zahorodniuk V. On isomorphisms of algebras of entire symmetric functions on Banach spaces. J. Math. Anal. Appl., 2023, Article Number 127370. doi:10.1016/j.jmaa.2023.127370.