Entire functions that share 0 with their shifts and difference operators

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The uniqueness of meromorphic functions sharing values with their shifts or difference operators has been investigated by many authors, see e.g. [1–4], we define its shift by $f_c(z) = f(z+c)$ and its difference operators by

$$\Delta_c f(z) = f(z+c) - f(z), \Delta_c^n f(z) = \Delta_c^{n-1}(\Delta_c f(z)), n \in \mathbb{N}, n \geq 2.$$

In this work, we investigate the uniqueness of an entire function $f(z)$ sharing one finite value a CM with $f(z+c)$ and $\Delta_c^2 f(z)$. In this case we find that $f(z) = h(z)e^{\frac{\alpha}{c}z}$ where $\alpha \neq 0$ and $h(z)$ is periodic entire function of period c. Here, we say that two entire functions $f(z)$ and $g(z)$ share a value a CM if $f(z) - a$ and $g(z) - a$ have the same zeros with same multiplicities.

Theorem 1. [3] *Let $f(z)$ be a transcendental entire function of finite order such that $f(z) \not\equiv f(z+c)$. Then $f(z)$, $f(z+c)$ and $\Delta_c f(z)$ can not share any finite value $a \neq 0$ CM. Furthermore, if $a = 0$, $f(z)$ must be of the following form $f(z) = h(z)e^{\frac{\alpha}{c}z}$, where $\alpha \neq 0$ and $h(z)$ is periodic entire function of period c.*

Theorem 2. *Let $f(z)$ be a transcendental entire function of finite order such that $f(z) \not\equiv f(z+c)$. Then $f(z)$, $f(z+c)$ and $\Delta_c^2 f(z)$ can not share any finite value $a \neq 0$ CM. Furthermore, if $a = 0$, $f(z)$ must be of the following form $f(z) = h(z)e^{\frac{\alpha}{c}z}$, where $\alpha \neq 0$ and $h(z)$ is periodic entire function of period c.*

Example 1. The entire function $f(z) = \exp(z)$ such that $f(z+1) = ef(z)$, $\Delta_1^2 f(z) = (e-1)^2 f(z)$, and hence $f(z)$, $f(z+1)$ and $\Delta_1^2 f(z)$ share 0 CM.

Theorem 3. *Let $f(z)$ be a transcendental entire function of finite order such that $f(z) \not\equiv f(z+c)$. Then $f(z), \Delta_c^2 f(z)$ and $\Delta_c^2 f(z+c)$ can not share any finite value $a \neq 0$ CM. Furthermore, if $a = 0$, $f(z)$ must be of the following form $f(z) = h(z)e^{\frac{\alpha}{c}z}$, where $\alpha \neq 0$ and $h(z)$ is periodic entire function of period c.*

Example 2. The entire function $f(z) = \exp(z)$ such that $\Delta_1^2 f(z) = (e-1)^2 f(z)$, $\Delta_1^2 f(z+1) = e(e-1)^2 f(z)$, and hence $f(z)$, $\Delta_1^2 f(z)$ and $\Delta_1^2 f(z+1)$ share 0 CM.

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