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The CLR Property allows the use of C-Class and Inverse C-Class Functions to establish a Common Fixed Point Theorem in Intuitionistic Menger Space

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This paper aims to establish the existence and uniqueness of a common fixed point for four self-mappings in Intuitionistic Menger metric spaces, subject to certain conditions that involve the (CLR) property and C-class functions. To support the validity of our hypotheses, we provide illustrative examples.

Our results extend several works, including [1], [2].

Definition 1. The pair (A, S) of self mappings of an intuitionistic Menger space $(X, F, L, *, \diamond)$ is said to have the common limit range property with respect to the mapping S (denoted by (CLR_S)) if there exists a sequence $(x_n) \subset X$ such that,

$$\lim_{n\to+\infty} Ax_n = \lim_{n\to+\infty} Sx_n = z \quad \text{where } z \in S(X).$$

Two pairs (A, S) and (B, T) of self mappings of an intuitionistic Menger space $(X, F, L, *, \diamond)$ are said to have the common limit range property with respect to mappings S and T (denoted by (CLR_{ST})) if there exists two sequences $\{x_n\}$, $\{y_n\} \subset X$ such that

$$\lim_{n\to+\infty} Ax_n = \lim_{n\to+\infty} Sx_n = \lim_{n\to+\infty} By_n = \lim_{n\to+\infty} Ty_n = z, \text{ where } z \in S(X) \cap T(X).$$

In 2014 the concept of C-class functions was introduced by A. H. Ansari [2], defined as

Definition 2. [2] We say that the continuous function $f : [0, +\infty)^2 \to \mathbb{R}$ is C-class function if the following assumptions satisfies for all $s, t \in [0, +\infty)$

- $f(s, t) \leq s$;
- $f(s, t) = s$ implies that $s = 0$ or $t = 0$.

We will denote the set of all C-class functions by $\mathcal{C}$.

Definition 3. [2] We say that the continuous function $g : [0, +\infty)^2 \to \mathbb{R}$ is inverse C-class function if the following assumptions satisfies for all $s, t \in [0, +\infty)$

- $g(s, t) \geq s$;
- $g(s, t) = s$ implies that $s = 0$ or $t = 0$.

We will denote the set of all inverse C-class functions by $\mathcal{C}_{inv}$.

Definition 4. [2] We say that the continuous function $\psi : [0, +\infty) \to [0, +\infty)$ is an altering distance function, if the following assumptions satisfies

- ψ is non-decreasing on $[0, +\infty)$;

- $\psi(t) = 0$ iff $t = 0$.

We shall denote the class of altering distance functions by Ψ.
Alternatively, the continuous function $\varphi : [0,1] \to [0,1]$ is also called an altering distance function, if the following assumptions satisfies

- φ is decreasing on $[0,1]$;
- $\varphi(t) = 0$ iff $t = 1$.

We shall denote the set of such functions by Φ.

Theorem 1. *Let A, B be self mappings of an intuitionistic Menger metric space $(X, F, L, *, \diamond)$ satisfying the following conditions*

1. *The pair (A, B) satisfies the (CLR_B) property;*
2. *$A(X) \subseteq B(X)$;*
3. *$B(X)$ is a closed subset of X;*
4. *$B(y_n)$ converges for every sequence $\{y_n\}$ in X whenever $A(y_n)$ converges;*
5. $$\psi\big(F_{Ax,By}(t)\big) \geq g\big(\psi(M(x,y,t)); \varphi(M(x,y,t))\big),$$
   and
   $$\psi\big(L_{Ax,By}(t)\big) \leq f\big(\psi(N(x,y,t)); \varphi(N(x,y,t))\big),$$
   where $\psi \in \Psi$, $\varphi \in \Phi$, and $f \in \mathcal{C}$, $g \in \mathcal{C}_{inv}$,
   $$M(x,y,t) = \min\{F_{x,y}(t), F_{Ax,x}(t), F_{By,y}(t), F_{x,By}(t), F_{y,Ax}(t)\},$$
   $$N(x,y,t) = \max\{L_{x,y}(t), L_{Ax,x}(t), L_{By,y}(t), L_{x,By}(t), L_{y,Ax}(t)\},$$
   for all $x, y \in X$,$t > 0$;
6. *(A, B) are weakly compatible.*

Then, the mappings A, B have a unique fixed point.

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2. Ansari A. H., Popa V., Singh Y. M. and Khan M. S. Fixed point theorems of an implicit relation via C-class function in metric spaces. J. Adv. Math. Stud, 2020, 13(1), 1–10.