A disjoint hypercyclicity criterion with applications to Toeplitz operators

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Let X be a topological vector space, $m \in \mathbb{N}$ and $(T_1, \dots, T_m) \in B(X)^m$. The m-tuple $(T_1, \dots, T_m)$ is called disjoint hypercyclic (d-hypercyclic) if there exists $x \in X$ such that

$$\{(T_1^n x, \dots, T_m^n x) : n = 0, 1, \dots\} \text{ is dense in } X^m.$$

The new concept of disjoint hypercyclic operator is derived from the concept of hypercyclic operator in linear dynamics. Recall that $x \in X$ is called a hypercyclic vector for a bounded linear operator T on X if $\{T^n x : n = 0, 1, \dots\}$ is dense in X and T is called hypercyclic if it has a hypercyclic vector. Obviously, $(T_1, \dots, T_m)$ is disjoint hypercyclic if and only if the operator $T_1 \oplus \dots \oplus T_m$ has a hypercyclic vector of the shape $(x, \dots, x)$ for some $x \in X$ [4]. Disjoint hypercyclicity was independently introduced in 2007 by Bernal and Gonzalez [1] and Bès and Peris [2].

In this talk, we will first provide a criterion to construct disjoint hypercyclic operators. This criterion is a d-hypercyclic version of the Godefroy-Shapiro Criterion for hypercyclic operators [3, p. 251, Theorem 4.5].

Theorem 1 (A d-hypercyclic version of the Godefroy-Shapiro Criterion)**.** *Let T_1 and T_2 be two distinct operators on a Banach space X. Suppose that the subspaces*

$$X_0 := span\{x \in X : T_1 x = \lambda_1 x \text{ and } T_2 x = \lambda_2 x \text{ for some } \lambda_1, \lambda_2 \in \mathbb{C}, |\lambda_1| < 1, |\lambda_2| < 1\}$$

$$Y_1 := span\{x \in X : T_1 x = \lambda_1 x \text{ and } T_2 x = \lambda_2 x \text{ for some } \lambda_1, \lambda_2 \in \mathbb{C}, |\lambda_1| > \max\{1, |\lambda_2|\}\}$$

and

$$Y_2 := span\{x \in X : T_1 x = \lambda_1 x \text{ and } T_2 x = \lambda_2 x \text{ for some } \lambda_1, \lambda_2 \in \mathbb{C}, |\lambda_2| > \max\{1, |\lambda_1|\}\}$$

are dense in X. Then T_1 and T_2 are d-hypercyclic.

We second will deal with the disjoint hypercyclicity of Toeplitz operators on the Hardy space $H^2(\mathbb{D})$ in the unit disc $\mathbb{D}$. Recall that for any $\psi \in L^\infty(\mathbb{T})$, the Toeplitz operator T_ψ with the symbol ψ is defined as

$$T_\psi f = P(\psi f),$$

where P is the orthogonal projection from $L^2(\mathbb{T})$ onto H^2. Toeplitz operators play an important role in operator theory and function theory of complex variables and have many applications in various areas. As the question for hypercylic Toeplitz operators asked by Shkarin [4], it is natural to ask the question for d-hypercyclic Toeplitz operators:

"Characterize d-hypercyclic Toeplitz operators $T_{\phi_1}, T_{\phi_2}, \dots, T_{\phi_N}$ in terms of the symbols $\phi_1, \phi_2, \dots, \phi_N \in L^\infty(\mathbb{T})$."

Using Theorem 1, we will construct some examples of disjoint hypercyclic operators for Toeplitz operators on $H^2(\mathbb{D})$. By motivated from [3], we will give some conditions for d-hypercyclicity of the antianalytic Toeplitz operators on $H^2(\mathbb{D})$. The second result of this study is the d-hypercyclicity of the antianalytic Toeplitz operators in terms of their symbols as follows.

Theorem 2. *Let φ_1 and φ_2 be two distinct nonconstant functions in H^∞. Suppose that the antianalytic Toeplitz operators $T_{\bar{\varphi}_1}$ and $T_{\bar{\varphi}_2}$ are d-hypercyclic. Then $\varphi_1(\mathbb{D}) \cap \mathbb{T} \neq \emptyset$ and $\varphi_2(\mathbb{D}) \cap \mathbb{T} \neq \emptyset$. Conversely, if $\varphi_1^{-1}(\mathbb{D}) \cap \varphi_2^{-1}(\mathbb{D})$, $\{z \in \mathbb{D} : |\varphi_1(z)| > \max\{1, |\varphi_2(z)|\}$ and $\{z \in \mathbb{D} : |\varphi_2(z)| > \max\{1, |\varphi_1(z)|\}$ are nonempty, then $T_{\bar{\varphi}_1}$ and $T_{\bar{\varphi}_2}$ are d-hypercyclic.*

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