Asymptotic behaviour of solutions of stochastic differential equations

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Let X be a solution of an n-dimensional ($n \geq 2$) stochastic differential equation

$$\mathrm{d}X(t) = a(X(t))\mathrm{d}t + b(X(t))\mathrm{d}W(t).$$

Define the *radius* R and the *angle* Φ of the process X as follows:

$$R(t) := |X(t)|, \qquad \Phi(t) := \frac{X(t)}{|X(t)|}.$$

We have studied the asymptotic behaviour of the radius and the angle of the process X as $t \to \infty$, namely, we have stated sufficient conditions such that almost surely:

- the radius is *transient*, i.e.,
$$\lim_{t\to\infty} R(t) = \infty;$$
- the angle *stabilizes*, i.e.,
$$\exists \lim_{t\to\infty} \Phi(t) =: \Phi_\infty;$$
- there is a deterministic asymptotic of the radius, i.e.,
$$\exists r\colon R(t) \sim r_{\Phi_\infty}(t),\ t \to \infty.$$