On queueing system with batch arrivals and disasters

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In this work we analyze an $M^{[X]}(\lambda)/M(\mu)/c$ queueing system. We assume that the times of arrivals are given by a Poisson process. The arrival batch size X is a random variable with probability mass function $\mathbb{P}(X = l) = b_l$, $l = 1, 2, \ldots$. They are served in accordance with First Come First Served (FCFS) discipline. After getting a service a customer may come back to the system for another service (if the former one is not satisfactory) with some probability β' and may decide to not return with a probability β. During the busy period the system may break down. At this time all present customers are removed out and the system (all the servers as one station) is sent for a reparation. The inter-arrival times between successive breakdowns are assumed to be distributed exponentially with rate η. Repair times have an exponential distribution with the same rate ϑ. On arrival, if a batch of customers find the c servers busy, they may decide to enter the system with a certain probability θ, or balk wit a complementary probability $\theta' = 1 - \theta$. More precisely, if we suppose that the number of customers in the batch is $n(\geq 1)$, then all customers of arrival batch join the system with probability θ if $n < c$, and all leave the system without receiving service (balk) with probability θ' otherwise. During the reparation of the primary servers customers can be served by substitute servers. The service times during this period are supposed to be exponentially distributed with a rate ν, where $\nu < \mu$. During this period new customers continue to enter the system. Once the reparation of the servers is finished, the service during this period is immediately stopped and the principal servers restart and operate at their service rate. In addition, once the system is repaired and the queue is empty, the primary servers return as one to the system, remain idle and wait for new arrivals. During a repair period customers can get impatient. Each customer activates an impatient timer T, exponentially distributed with rate χ. If the customer has not been served before its impatience time has expired once he leave the system without getting a service. The inter-arrival times, repair times, impatience times, service times are supposed to be mutually independent.

Let $N(t)$ be the number of customers in the system and $J(t)$ denote the status of the server at time t. If $J(t) = 1$, the system is functioning serving customers, whereas if $J(t) = 2$, the system is down undergoing a repair process. Then $\{(N(t), J(t)); t \geq 0\}$ represent two-dimensional infinite state continuous-time Markov chain with state space $\mathcal{S} = \{(n, j) : n \geq 0, j = 1, 2\}$. Let $\pi_{n,j} = \lim_{t\to\infty} \pi\{N(t) = n, J(t) = j\}$, $n \geq 0$, $j = 1, 2$, define the invariant state probabilities of the process $\{(N(t), J(t);), t \geq 0\}$. Using the theory of Markov processes we show that the steady-state equations of the model are the following.

1. $J(t) = 1$ **(normal busy period)**

$$\begin{aligned}
\lambda\pi_{0,1} &= \vartheta\pi_{0,2} + \beta\mu\pi_{1,1}, \ \ n = 0, \\
(\lambda + \eta + \beta\mu)\pi_{1,1} &= \lambda b_1\pi_{0,1} + \vartheta\pi_{1,2} + 2\beta\mu\pi_{2,1}, \ \ n = 1, \\
(\lambda + \eta + n\beta\mu)\pi_{n,1} &= \lambda\sum_{m=1}^{n} b_m\pi_{n-m,1} + \vartheta\pi_{n,2} + (n+1)\beta\mu\pi_{n+1,1}, \\
& \qquad\qquad 2 \le n \le c-1, \\
(\theta\lambda + \eta + n\beta\mu)\pi_{n,1} &= \lambda\sum_{m=1}^{n} b_m\pi_{n-m,1} + \vartheta\pi_{n,2} + c\beta\mu\pi_{n+1,1}, \ \ n = c, \\
(\theta\lambda + \eta + c\beta\mu)\pi_{n,1} &= \theta\lambda\sum_{m=1}^{n} b_m\pi_{n-m,1} + \vartheta\pi_{n,2} + c\beta\mu\pi_{n+1,1}, \ \ n \ge c.
\end{aligned}$$

2. $J(t) = 2$ **(working breakdown period)**

$$\begin{aligned}
(\lambda + \vartheta)\pi_{0,2} &= \eta\sum_{n=1}^{\infty}\pi_{n,1} + (\beta\nu + \chi)\pi_{1,2}, \ \ n = 0, \\
(\lambda + \vartheta + \beta\nu + \chi)\pi_{1,2} &= \lambda b_1\pi_{0,2} + 2(\beta\nu + \chi)\pi_{n+1,2}, \ \ n = 1, \\
(\lambda + \vartheta + n(\beta\nu + \chi))\pi_{n,2} &= \lambda\sum_{m=1}^{n} b_m\pi_{n-m,2} + (n+1)(\beta\nu + \chi)\pi_{n+1,2}, \\
& \qquad\qquad 2 \le n \le c-1, \\
(\theta\lambda + \vartheta + n(\beta\nu + \chi))\pi_{n,2} &= \lambda\sum_{m=1}^{n} b_m\pi_{n-m,2} + (c\beta\nu + (n+1)\chi)\pi_{n+1,2}, \\
& \qquad\qquad n = c, \\
(\theta\lambda + \vartheta + c\beta\nu + n\chi)\pi_{n,2} &= \theta\lambda\sum_{m=1}^{n} b_m\pi_{n-m,2} + (c\beta\nu + (n+1)\chi)\pi_{n+1,2}, \\
& \qquad\qquad n \ge c.
\end{aligned}$$