Asymptotics related to number of upcrossings of a Gaussian random process

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Given a Gaussian centered random field $\xi : \mathbb{R}^2 \to \mathbb{R}$ with the covariance $cov(\xi(u), \xi(v)) = e^{-||u-v||^2}$, we define the random process $\eta(t) = \xi(f(t))$, where $f : [0,1] \to \mathbb{R}^2, f \in C_p^2([0,1])$, is a parametrization of a closed smooth curve without self-intersections. As an example, consider the function $f(t) = (R\cos t, R\sin t)$. Then $\eta_R(t) = \xi(R\cos t, R\sin t)$. The covariance of the process η_R equals

$$E\eta_R(t)\eta_R(s) = e^{-R^2(2-2cos(t-s))}.$$

It can be seen that $E\eta_R(t)\eta_R(s) \to 0, R \to \infty$. To obtain a non-trivial limit we consider the process

$$\zeta_R(t) = \eta_R\left(\frac{t}{R}\right).$$

Then

$$E\zeta_R(t)\zeta_R(s) = e^{-R^2(1-cos\frac{t-s}{R})} \to e^{-(t-s)^2}, R \to \infty.$$

In the limit we obtain a covariance of a stationary Gaussian process ζ_∞.

Definition 1. The number of upcrossings of a given level $c > 0$ by a random process η in time T is

$$N_c(T) = \#\{t \in [0,T] : \eta'(t) > 0, \eta(t) = c\}$$

We can use the Rice formula to calculate the number of upcrossings by a process of level c in time T by a stationary process [1]:

$$EN_c(T) = T\int_0^\infty yp(c,y)dy,$$

where p is the joint density of the process and its derivative at a fixed time.

In the case of ζ_∞ we have

$$EN_c(T) = Te^{-c^2/2}.$$

We address the following question: how the sequence of levels c_m must increase in order to get a finite limit of $N_c(T)$? We use a discrete partition of $[0,T]$ into n subsegments and study the distribution of the sum

$$\sum_{k=0}^{n} I\left\{\zeta_\infty\left(\frac{kT}{n}\right) > c_m\right\}.$$

1. Adler R. J., Taylor J. E. Random Fields and Geometry. — Springer Monographs in Mathematics, 2007, 450.