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Univariate Compound Models Based on Random Sum of Variates with Related Application

M. A. Meraou[1], N. M. Al-Kandari[2], M. Z. Raqab[3]

[1]Laboratory of Statistics and Stochastic Processes, University of Djillali Liabes, BP 89, Sidi Bel Abbes 22000, Algeria

[2]Department of Statistics and OR, Kuwait University, Safat 13060, Kuwait

[3]Department of Mathematics, The University of Jordan, Amman 11942, Jordan

amine.meraou@univ-sba.dz, noriah.alkandari@ku.edu.kw, mraqab@ju.edu.jo

The idea of compounding is quite common in the statistical literature. The compound models can be used in many fields such as hydrology, risk model, financial economics studies (see, e.g. [1]), and environmental studies (see, e.g. [2]). The compound Poisson model is very flexible and it can be used efficiently to model different types of data sets. For this reason, considerable attention has been devoted to the problem of the best fit for the experimental data. It is well documented in the literature that the Gamma distribution plays a pivotal role for modeling the claim severity in actuarial science and it can be used quite effectively to analyze skewed data. Compound models have been used quite extensively in analyzing insurance losses data, one may refer to [3] and [4]. In this work, we develop a new version of skewed distribution based on modeling aggregate claims. Let N and M^* (conditioning that $M > 1$) denote the total number of claims and approved claims, respectively, in a fixed time period, and $X_1, \ldots, X_{M^*}$ denote the amount paid by the insurance company. Hence $Y = X_1 + \ldots + X_{M^*}$ is the aggregate amount paid by the insurance company in that fixed time period. It is assumed that X_i's are independent identically distributed (i.i.d) Gamma random variables with parameters α and β, and M^* is a zero-truncated Poisson random variable with parameter θ, independent from X_i's. We call this new distribution as a univariate compound zero-truncated Poisson Gamma (UZTP-GA) distribution. The Gamma distribution can be obtained as a special case of the proposed compound UZTP-GA distribution.

Suppose $\{X_1, X_2, \ldots\}$ is a sequence of i.i.d Gamma random variables with shape parameter α and scale parameter β (GA(α,β)). Let the random variable $M^\star$ has a zero-truncated Poisson distribution with parameter θ :

$$P(M^* = m) = \frac{\theta^m}{m!(e^\theta - 1)};\ m = 1, 2, 3, \ldots,\ \theta > 0.$$

The random variable Y with the stochastic representation $Y \overset{D}{=} \sum_{i=1}^{M^\star} X_i$ is said to follow a UZTP-GA model with parameters α, β and θ. This model is denoted by UZTP-GA(α,β,θ).

The cumulative density function of Y can be obtained as follows

$$F_Y(y) = P(Y \le y) = \frac{1}{e^\theta - 1}\sum_{m=1}^{\infty}\frac{\Gamma_{\beta y}(m\alpha)}{\Gamma(m\alpha)}\cdot\frac{\theta^m}{m!}.$$

The probability density function of Y can be written as

$$f_Y(y) = \frac{e^{-\beta y}}{y(e^\theta - 1)}\sum_{m=1}^{\infty}\frac{[(\beta y)^\alpha\theta]^m}{m!\ \Gamma(m\alpha)};\ y > 0,\ \alpha,\beta,\theta > 0.$$

Table 1: The AE and the associated MSEs of the MLEs for the UZTP-GA model

n	Par	Set 1		Set 2		Set 3	
		AE	MSE	AE	MSE	AE	MSE
25	α	0.5692	0.0317	1.1226	0.0512	1.8712	0.0878
	β	1.2483	0.1627	1.2965	0.1795	1.7997	0.2341
	θ	0.5932	0.0327	0.7214	0.0916	1.2261	0.1119
50	α	0.5406	0.0141	1.1135	0.0331	1.8550	0.0486
	β	1.1573	0.1072	1.2162	0.1082	1.7572	0.1409
	θ	0.5801	0.0182	0.6443	0.0799	1.2158	0.0859
100	α	0.5362	0.0075	1.1136	0.0248	1.8392	0.0265
	β	1.1223	0.0548	1.1862	0.0686	1.7315	0.0862
	θ	0.5641	0.0098	0.6148	0.0403	1.2086	0.0533

Table 2: The goodness of fit tests for insurance data set

Model	AIC	AICc	HQIC	BIC	K-S	p-value
UZTP-GA	374.842	375.616	376.452	379.508	0.1432	0.4716
CG-GA	377.442	378.216	379.052	382.108	0.1516	0.3990
GA	377.951	378.326	379.025	381.062	0.1602	0.2975
EXP	379.748	379.870	380.285	381.304	0.1902	0.1390
GA-EXP	374.514	375.847	376.661	380.735	0.1374	0.4812
Lindley-EXP	377.580	378.354	379.190	382.246	0.1517	0.3591

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