Transient phenomena for total progeny in Galton-Watson processes with immigration

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Consider critical Galton-Watson branching processes with immigration. It is known [1] that if the second moment B of the generating function for single particle progeny is finite, as well as the mathematical expectation β of the immigration component, then the following convergence takes place for the total number of individuals Y_n:

$$E\left(exp\left\{\ Y_n\beta/n^2\ \right\}\right) \xrightarrow{n\to+\infty} \left(ch\sqrt{B\beta/2}\right)^{2\beta/B}.$$

Now consider processes that are close to critical, meaning that the mathematical expectation A of singular individual progeny approaches unity as n approaches infinity. We prove that if $0 < \lim\limits_{A\to1} B = \lim\limits_{A\to1} B(A) = \tilde{B} < +\infty$ and $0 < \lim\limits_{n\to\infty, A\to1} \beta = \lim\limits_{n\to\infty, A\to1} \beta(A,n) = \tilde{\beta} < +\infty$, then the next theorem takes place:

Theorem 1.

$$E\left(exp\left\{\ Y_n\theta/E(Y_n)\ \right\}\right) \xrightarrow{n\to\infty, A\to1} \begin{cases} \left(ch\sqrt{\tilde{B}\theta/\tilde{\beta}}\right)^{2\tilde{\beta}/\tilde{B}}, \lim\limits_{n\to\infty, A\to1} n(1-A) = 0; \\ G^{\pm}(\theta,w), 0 \neq \lim\limits_{n\to\infty, A\to1} n|1-A| = w < +\infty; \\ \left(1+\theta\tilde{B}/2\tilde{\beta}\right)^{-2\tilde{\beta}/\tilde{B}}, \lim\limits_{n\to\infty, A\to1} n(1-A) = -\infty; \\ e^{-\theta}, \lim\limits_{n\to\infty, A\to1} n(1-A) = +\infty, \end{cases}$$

where $G^{\pm}(\theta,w) = \left(e^{\pm\frac{w}{2}} \frac{e^{\frac{w}{2}\sqrt{1+\frac{2\tilde{B}\theta}{\tilde{\beta}U(w)}}} + e^{-\frac{w}{2}\sqrt{1+\frac{2\tilde{B}\theta}{\tilde{\beta}U(w)}}}}{1+c}\right)^{2\tilde{\beta}/\tilde{B}}$, $U(w) = e^{\pm w} - 1 \mp w,\ c = \frac{\sqrt{1+\frac{2\tilde{B}\theta}{\tilde{\beta}U(w)}}\mp1}{\sqrt{1+\frac{2\tilde{B}\theta}{\tilde{\beta}U(w)}}\pm1}$.
Here $G^{+}(\theta,w)$ *corresponds to the case* $A \searrow 1$ *and* $G^{-}(\theta,w)$ *corresponds to the case* $A \nearrow 1$.

1. Pakes A. G. Further results on the critical Galton-Watson process with immigration. Journal of the Australian Mathematical Society, 1972, 13, 3, 277-290.