ON THE ASYMPTOTIC BEHAVIOR OF ONE CLASS OF TWO-DIMENSIONAL MARKOV CHAINS

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In this work we are interested to study the asymptotic behavior of the Markov chain $(Z_n)_{n\geq 0}$ on $[0,1]\times[0,1]$ corresponding for example to the successive positions of a robot in a square room. Specifically, if the robot is at $x=(x_1,x_2)\in[0,1]\times[0,1]$ at time n, it selects at time $n+1$ one of the four rectangles $[0,x_1]\times[0,x_2]$, $[0,x_1]\times[x_2,1]$,$[x_1,1]\times[0,x_2]$ or $[x_1,1]\times[x_2,1]$ with probabilities $p_{00}(x_1,x_2)$, $p_{01}(x_1,x_2)$, $p_{10}(x_1,x_2)$ and $p_{11}(x_1,x_2)$ and then moves to a random point $y=(y_1,y_2)$ in the chosen rectangle.

We assume that the functions $p_{ij}, 0\leq i,j\leq 2$, are continuous and non negative on $[0,1]\times[0,1]$, and satisfy $p_{00}(x_1,x_2)+p_{01}(x_1,x_2)+p_{10}(x_1,x_2)+p_{11}(x_1,x_2)=1$.

The chain $(Z_n)_{n\geq 0}$ fits into the framework of iterated random functions with place dependent probabilities. By using quasi-compact linear operators technique (see [1], [2] and [3]) we prove the following theorem which give the sufficient condition for the uniqueness of the stationary probability measure.

Let Q be the transition operator of the chain $(Z_n)_{n\geq 0}$, and we denote $\mathcal{H}_\alpha([0,1]\times[0,1])$, the space of α-Hölder continuous functions from $[0,1]\times[0,1]$ to $\mathbb{C}$.

Theorem 1. *Assume that*

1. *for all $i,j=0,1$, the functions p_{ij} belong to $\mathcal{H}_\alpha([0,1]\times[0,1])$.*
2. *there exist $i,j\in\{0,1\}$ such that $\delta_{ij}=\min_{x\in[0,1]\times[0,1]} p_{ij}(x)>0$.*

Then, the Diaconis-Freedman chain on $[0,1]\times[0,1]$ has a unique Q-invariant probability measure ν. Furthermore, there exist constants $\kappa>0$ and $\rho\in[0,1[$ such that

$$\forall\varphi\in\mathcal{H}_\alpha([0,1]\times[0,1]),\ \forall x\in[0,1]\times[0,1]\quad |Q^n\varphi(x)-\nu(\varphi)|\leq\kappa\rho^n.$$

The condition 2 of the Theorem 1 is a sufficient condition to ensure the unicity of an invariant probability measure but it is clearly too strong. In this work, we propose a complete description of the invariant measure for this chain when condition 2 is not satisfied and explore the case when there are several invariant measures.

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