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Quadratic entropy

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Let H be a real separable Hilbert space with inner product $\langle \cdot\ ,\ \cdot \rangle$ and norm $\| \cdot \|$. The quadratic entropy of a set $A \subset H$ is defined as follows. For any infinite or finite sequence $\{u_n\}$ in A consider the orthogonal complement πu_j of the vector u_j onto the linear span of all vectors $\{u_n\}$ except u_j itself.

Definition 1. Quadratic entropy of a set $A \subset H$ is defined as

$$H(A) = \sup \sum_n \|\pi u_j\|^2,$$

where the supremum is taken over all possible finite or infinite sequences $\{u_n\}$ in A.

This definition was given in [1] in order to study certain properties of Brownian stochastic flows. For the space of Gaussian random variables we can redefine $\|\pi u_j\|^2$ in terms of the conditional variance, as

$$\|\pi u_j\|^2 = Var(u_j | u_i, i \neq j).$$

Example 1. For a ball $B(0,1)$ in $\mathbb{R}^n$ we can precisely calculate the quadratic entropy:

$$H(B(0,1)) = n.$$

We study the quadratic entropy of the set $\{\xi(t) : t \in [0,1]\}$, where ξ is a Gaussian process on $[0,1]$ with zero mean and the covariance function $E\xi(s)\xi(t) = \exp(-\mid s-t \mid^{\alpha})$, $\alpha \in [1,2]$.

We provide a lower bound using results of [2]:

$$\|\pi \xi(t_j)\|^2 \geq \frac{C_1}{n^{\alpha+1}}.$$

1. Dorogovtsev A. The entropy of stochastic flow. Sbornik: Mathematics, 2010, 201, 5, 645.
2. Cuzick J., DuPreez J. P.. Joint continuity of Gaussian local times. The Annals of Probability, 1982, 10, 3, 810-817.