Modified bisection algorithm in estimating the extreme value index

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The General Pareto Distribution (GPD) has been widely used a lot in the extreme value theory, for example, to model exceedance over a threshold. When applied to real data sets the GPD depends substantially and clearly on the parameter estimation process. The most used estimation is the maximum likelihood. In this paper we give a new estimation method for the maximum likelihood estimate (MLE) of the extreme value index. The study of extreme values relates to the study of the tail index of the distribution function. Also, the GPD is based on the tail index. From a statistical perspective, the threshold is loosely defined in such a way that the population tail can be well approximated by an extreme value model (e.g., the GPD). Then we can use the MLE of the GPD via the likelihood approach for estimating extreme value index. The aim of this study is the improvement of the algorithm that deal with GPD estimation via the MLE given by Kouider [2] when $\gamma \geq -1$. The likelihood equations for GPD are given in terms of the partial derivatives that have been studied in many articles including the important works [1,2]. The resulting likelihood equations of GPD in terms of the excesses $C_j = X_j - t$ are as follows

$$\left\{ \begin{array}{ll} \psi(\theta) = \left(\frac{1}{k} \sum_{j=1}^{k} \log(1 - \theta C_j) + 1 \right) \cdot \left(\frac{1}{k} \sum_{j=1}^{k} (1 - \theta C_j)^{-1} \right) - 1 = 0 & \text{for } \gamma \neq 0 \\ \widehat{\sigma}^{ML} = 1/\overline{C} & \text{for } \gamma = 0 \end{array} \right\}, \tag{1}$$

where $\theta = -(\gamma/\sigma)$ and $\psi(\theta)$ is defined on $\mathcal{B} = \{\theta \in [\theta_L, -\varepsilon] \cup [\varepsilon; \theta_U],\ \theta \neq 0\}$ with $\theta_U = 1/C_{k,k} - \varepsilon$, $\theta_L = 2(C_1)^{-2}\left(C_1 - \overline{C}\right)$, $\varepsilon = 10^{-p}/\overline{C}$, p being a natural number (see the Theorem 1 in [2]).

Then for $t = X_{n-k,n}$ the MLE of GPD can be approximated in the following procedure:

1. Find the root $\widehat{\theta}$ of $\psi(\theta) = 0$ from the equation (1).

2. Compute $\widehat{\gamma}^{ML} = \frac{1}{k} \sum_{j=1}^{k} \log\left(1 - \widehat{\theta} C_j\right)$.

3. Set $\widehat{\sigma}^{ML} = -\left(\widehat{\gamma}^{ML}/\widehat{\theta}\right)$.

To find the solution of $\psi(\theta) = 0$ in (1) we use the Modified Bisection Algorithm (MBA) for multiple roots that was proposed in [2]. The method is based on improved bisection and false position for multiple roots, and is realized as an R program. Next, to determine $\widehat{\gamma}^{ML}$ and $\widehat{\sigma}^{ML}$ we use the algorithm for the MLE presented in [2] for the GPD parameters (γ, σ). For numerical example we consider confidence bounds for the monthly maximal losses of the Danish data. Using the algorithm for the MLE of the GPD parameters (γ, σ) over the threshold $k > 50$, we find the GPD maximum likelihood estimates $\widehat{\gamma} = 0.08586831$ and $\widehat{\sigma} = 32.31397400$. Corresponding 95% confidence intervals are $-0.1249036 < \gamma < 0.2966402$ and $21.08807 < \sigma < 43.53988$.

1. de Haan L., Ferreira A. Extreme Value Theory: An Introduction. — New York: Springer, 2006, 418 p.
2. M. R. Kouider. On maximum likelihood estimates for the shape parameter of the generalized Pareto distribution. Science Journal of Applied Mathematics and Statistics, 2019, Vol. 7, No. 5, pp. 89-94.