The best and the worst Nash equilibrium and the anarchy price

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Congestion game is a class in the game theory, first proposed by the American economist R. Rosenthal in 1973. This type of game is noncooperatif. Each player chooses his resources and the payoff of each resource depends only on the number of players using it. The utility of each player depends on the number of players who choose the same or some overlapping strategy.

A Nash equilibrium is a term coined by the mathematician J. Nash in 1940. In this theory each player is assumed to know the equilibrium strategies of the other players, and no one has anything to gain by changing only one's own strategy.

In 1973 R. Rosenthal proved that any congestion game is a potential game, while Monderer and Shapley proved that for any potential game there is a congestion game with the same potential function. However, Konishi, Milchtaich and Quint observed that congestion games do not admit (in general) a potential function, but are likely to admit a Nash equilibrium in pure strategies.

In [1] a new class of congestion games was introduced under the name of congestion games with player-specific payoff function where each player has to choose only one resource, and he proved that any singleton congestion game always admit at least one Nash equilibrium. In 2010 Sbabou proved that any singleton congestion game has at least one equilibrium and developed a method to describe the Nash equilibria. The price of anarchy was originally defined by C.H. Papadimitriou to capture the worst case selfish performance of a simple game of N players that compete for M parallel links. We are interested in singleton congestion games. Our aim is to develop a method for calculating all Nash equilibria and we have developed an algorithm to calculate the best Nash equilibrium and the worst one. Then we calculate the price of anarchy.

1. Milchtaich I. Congestion games with player-specific payoff functions. Games and Economic Behavior, 1996, 13, 1 , 111-124