A note on asymptotic behavior of the overshoot distribution for a Lévy process

Ie. Karnaukh[1]

[1] Oles Honchar Dnipro National University, Dnipro, Ukraine

ievgen.karnaukh@gmail.com

The asymptotic behavior of overshoot for different classes of Lévy processes, conditional on making first passage, as the barrier tends to infinity, have been considered by many authors (see, e.g., [1-5]). Let $\{X_t, t \geq 0\}$ be a Lévy process with the cumulant

$$k(r) = ar + \lambda\left(\frac{c}{c-r} - 1\right), a < 0, \lambda, c > 0,$$

and with the asymptotic drift $m = \mathsf{E}X_1 = k'(0) = a + \frac{\lambda}{c}$. Set $\tau_x^+ = \inf\{t > 0 : X_t > x\}$ and $\gamma^+(x) = X_{\tau_x^+} - x, x \geq 0$. From [4, p. 37] we have

$$\mathsf{E}\left[e^{-s\tau_x^+ - u\gamma^+(x)}, \tau_x^+ < \infty\right] = \frac{c - \rho_+(s)}{c+u} e^{-\rho_+(s)x}, s, u > 0,$$

where $\rho_+(s)$ is the inverse of the cumulant $k(r)$ with the limit behavior

$$\lim_{s\downarrow 0} \rho_+(s) = \begin{cases} 0, & m \geq 0, \\ \rho_+, & m < 0, \end{cases} \qquad \rho_+ > 0,$$

Hence,

$$\mathsf{E}\left[e^{-s\tau_x^+ - u\gamma^+(x)} | \tau_x^+ < \infty\right] = \begin{cases} \frac{c-\rho_+(s)}{c+u} e^{-\rho_+(s)x}, & m \geq 0, \\ \frac{c-\rho_+(s)}{c-\rho_+} \frac{c}{c+u} e^{-(\rho_+(s)-\rho_+)x}, & m < 0. \end{cases}$$

After passage to the limit as $x \uparrow \infty$ and then $s \downarrow 0$:

$$\lim_{s\downarrow 0} \lim_{x\uparrow\infty} \mathsf{E}\left[e^{-s\tau_x^+ - u\gamma^+(x)} | \tau_x^+ < \infty\right] = \begin{cases} 0 & m \geq 0, \\ 0, & m < 0, \end{cases}$$

but

$$\lim_{x\uparrow\infty} \lim_{s\downarrow 0} \mathsf{E}\left[e^{-s\tau_x^+ - u\gamma^+(x)} | \tau_x^+ < \infty\right] = \begin{cases} \frac{c}{c+u} & m \geq 0, \\ \frac{c}{c+u}, & m < 0. \end{cases}$$

Thus, we should be careful when using the fluctuation identities to obtain $\mathsf{E}\left[e^{-\gamma^+(\infty)} | \tau_\infty^+ < \infty\right]$.

1. Kluppelberg C., Kyprianou A. E., Maller R. A. Ruin probabilities and overshoots for general Lévy insurance risk processes. The Annals of Applied Probability, 2004, 14, 4, 1766 – 1801.
2. Doney R. A., Kyprianou A. E. Overshoots and undershoots of Lévy processes. The Annals of Applied Probability, 2006, 16, 1, 91 – 106.
3. Kyprianou A. E., Pardo J. C., Rivero V. Exact and asymptotic n-tuple laws at first and last passage. The Annals of Applied Probability, 2010, 20, 2, 522 – 564.
4. Gusak D. Boundary functionals for Lévy processes and their applications. —Lambert Academic Publishing, 2014, 412 p.
5. Griffin P. S. Sample path behavior of a Lévy insurance risk process approaching ruin, under the Cramér–Lundberg and convolution equivalent condition. The Annals of Applied Probability, 2016, 26, 1, 360 – 401.