International Conference of Young Mathematicians
The Institute of Mathematics of the National Academy of Sciences of Ukraine
June 1–3, 2023, Kyiv, Ukraine

Intermittency Phenomena for Mass Distributions of Stochastic Flows with Interaction

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The intermittency phenomenon occurs when high peaks in a qunatity occur rarely but are signicant enough to influence the asymptotics of the underlying quantity. Mathematically speaking this can be characterised with moments. In this talk the topic of intermittency will be covered for mass distributions for stochastic differential equations with interaction, namely

$$\begin{cases} dx(u,t) &= a(x(u,t),\mu_t)\mathrm{d}t + b(x(u,t),\mu_t)\mathrm{d}B_t \\ x(u,0) &= u, \ u \in \mathbb{R}^d \\ \mu_t &= \mu_0 \circ x^{-1}(\cdot,t) \end{cases},$$

where μ_0 is a probability measure and $a : \mathbb{R}^d \times \mathfrak{M}(\mathbb{R}^d) \to \mathbb{R}^d$ and $b : \mathbb{R}^d \times \mathfrak{M}(\mathbb{R}^d) \to \mathbb{R}^{d\times d}$ are coefficients. Here $\mathfrak{M}(\mathbb{R}^d)$ is the space of all probability measures on $\mathbb{R}^d$ and $d \geq 1$ denotes the dimension. In the talk only measures that are absolutely continuous with respect to the Lebesgue measure will be investigated. The space of probability measures will be equipped with the Wasserstein distance

$$\gamma(\mu,\nu) = \inf_{\kappa\in C(\mu,\nu)} \int\int \frac{|u-v|}{1+|u-v|}\kappa(\mathrm{d}u,\mathrm{d}v) \tag{1}$$

Theorem 1. *Let a and b be Lipschitz continuous with respect to all arguments and continuously differentiable with respect to the spatial variable. Moreover assume that a and b are bounded. Then μ_t is almost surely absoulutely continuous with respect to the d-dimensional Lebesgue measure for all $t \geq 0$ with Lebesgue density:*

$$p_t = p_0(x^{-1}(\cdot,t))\det(Dx^{-1}(\cdot,t))$$

Then intermittency is defined as followed

Definition 1. $(p_t)_{t\geq 0}$ is intermittent if $(\frac{\lambda_p}{p})_{p\geq 1}$ is strictly increasing for

$$\lambda_p = \lim_{t\to\infty} \frac{\ln\left(\int_{\mathbb{R}^d} p_t^p(u)\mathrm{d}u\right)}{t}.$$

It turns out intermittency exists under dissipativity conditions on the coefficients.

Theorem 2. *Let a and b satisfy conditions of Theorem 1, assume furthermore that*

1. *there exists a continuously differentiable function ϕ such that ϕ and its derivative is bounded, and for all $u, v \in \mathbb{R}^d$*

$$(u-v, \phi(u)-\phi(v)) \leq -\alpha|u-v|^2$$

for some $\alpha > 0$;

2. *if B is the Lipschitz constant of b with respect to the spatial variable, then*

$$2\alpha - B^2(2q-1) > 0,$$

where $q > d$.

Then p_t is intermittent.