On the boundary local time of a reflected diffusion in space

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Let X be a Brownian motion, or more generally an a-symmetric diffusion, where a is the diffusion matrix,

$$X(t) = x + \int_0^t b(X_s)ds + \int_0^t \sigma(X_s)dB_s - \int_0^t \gamma_a(X_s)dL_s,$$

where B is a standard Brownian motion, L is the boundary local time and $b = \nabla a$ (recall that $a = \sigma\sigma^*$), reflecting in the direction of the (outer) co-normal γ_a in a smooth bounded domain $D \subset \mathbb{R}^d$, $d \geq 1$ (e.g. with a $\mathcal{C}^3$ boundary ∂D), with a $\mathcal{C}^2$ bounded and non-degenerate matrix a. For $t > 0$ and $n, k \in \mathbb{N}$ let $N(n,t)$ be the number of dyadic intervals $I_{n,k}$ of length 2^{-n}, $k \geq 0$, that contain a time $s \leq t$ such that $X(s) \in \partial D$. For a suitable normalizing factor $H(t)$ we prove, extending the one dimensional case [1], that a.s. for all $t > 0$ the entropy functional $N(n,t)/H(2^{-n})$ converges to the boundary local time $L(t)$ as $n \to \infty$. Applications include boundary value problems in PDE theory, efficient Monte Carlo simulations and Finance.

1. Laissaoui D., Benchérif Madani A. A limit theorem for local time and application to random sets. Stat. Prob. Letters, 2014, 88, 107-117.