GLOBAL WELL-POSEDNESS AND EXPONENTIAL DECAY OF FULLY DYNAMIC AND ELECTROSTATIC OR QUASI-STATIC PIEZOELECTRIC BEAMS SUBJECT TO A SOME DISTRIBUTED DELAY TYPES

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Piezoelectric materials, such as quartz, Rochelle salt, and barium titanate, have the property of converting from mechanical energy to electromagnetic energy, or of generating an internal electrical charge from applied mechanical pressure (see [5] for more details). The brothers, Pierre and Jacques Curie, first demonstrated the direct piezoelectric effect in 1880 [4], where single crystal quartz is the first material used in early experiments with piezoelectricity. These same materials, when exposed to electricity produce a relative tension. This phenomenon is known as the reverse piezoelectric effect. In [2] Ramos et al. they proved exponential stability for piezoelectric beams with magnetic effect, and dissipation produced by damping (δv_t) added to the first equation, in the case $V(t) = 0$, and by using finite differences method, they find numerical energy related to their system, where they used specific values. Also, Ramos et al. [1], they demonstrated the exponential stability for system of piezoelectric beams with delayed

$$\begin{cases} \rho v_{tt} - \alpha v_{xx} + \gamma\beta p_{xx} + \xi_1 v_t + \xi_2 v_t(x, t-\tau) = 0, & \text{in }]0, L[\times]0, +\infty[\,, \\ \mu p_{tt} - \beta p_{xx} + \gamma\beta v_{xx} = 0, & \text{in }]0, L[\times]0, +\infty[\,, \end{cases} \tag{1}$$

with the boundary and initial conditions

$$\begin{cases} v(0,t) = \alpha v_x(L,t) - \gamma\beta p_x(L,t) = 0, & t \geq 0, \\ p(0,t) = p_x(L,t) - \gamma v_x(L,t) = 0, & t \geq 0, \\ v_t(x, t-\tau) = f_0(x, t-\tau), & (x,t) \in]0, L[\times]0, \tau[\\ v(x,0) = v_0(x), v_t(x,0) = v_1(x), & x \in]0, L[\,, \\ p(x,0) = p_0(x),\ p_t(x,0) = p_1(x), & x \in (0, L)\,, \end{cases} \tag{2}$$

where $\xi_2 v_t(x, t-\tau)$ is the time of delay on vertical displacement, $\tau > 0$ is the respective retardation time, where they proved this stability under the conditions $\xi_1 > \xi_2$. Seghour et al. [3], they considered in $[0,1] \times [0,\infty)$ thermoelastic laminated system subject to a neutral delay. Under some conditions on the kernel h and some system parameters, they proved exponential and polynomial stability by using the energy method.

Motivated by the above works, in the present work, we consider the following fully dynamic piezoelectric system subject to a neutral delay

$$\begin{cases} \rho\left(v_t + \int_0^t k(t-s)\, v_t(s)\, ds\right)' - \alpha v_{xx} + \gamma\beta p_{xx} = 0, & \text{in } (0,L) \times (0,\infty)\,, \\ \mu p_{tt} - \beta p_{xx} + \gamma\beta v_{xx} = 0, & \text{in } (0,L) \times (0,\infty)\,, \\ v(0,t) = \alpha v_x(L,t) - \gamma\beta p_x(L,t) = 0, & t \geq 0, \\ p(0,t) = p_x(L,t) - \gamma v_x(L,t) = 0, & t \geq 0, \\ v(x,0) = v_0(x), v_t(x,0) = v_1(x), & x \in (0,L)\,, \\ p(x,0) = p_0(x),\ p_t(x,0) = p_1(x), & x \in (0,L)\,, \end{cases} \tag{3}$$

the initial data v_0, v_1, p_0, p_1 belongs to the suitable functional space and the convolution term involving the kernel k describes the neutral delay. The aim or novelty of the present work is to prove the global well-posedness of the system by using the Faedo-Galerkin method. Secondly, by constructing an appropriate Lyapunov functional using the multipliers method and without any damping term, we show that the neutral delay guarantees an exponential decay of the solution, irrespective of any condition on the wave speeds $\left(\sqrt{\frac{\alpha}{\rho}},\ \sqrt{\frac{\beta}{\mu}}\right)$ or any other condition on system parameters. Our results are related to some assumptions only on the kernel k of neutral delay term. Finally, the results are compared to the ones of the electrostatic case.

Theorem 1. *Let* $(v_0, v_1, p_0, p_1) \in H = \left[\hat{H}^1(0, L) \times L^2(0, L)\right]^2$. *Then the system (3) has a unique global strong solution and satisfies*

$$(v, p) \in C\left(\mathbb{R}_+,\ \hat{H}^2(0, L) \cap \hat{H}^1(0, L)\right) \cap C^2\left(\mathbb{R}_+,\ \hat{H}^1(0, L)\right). \tag{4}$$

The goal of this theorem is to prove the existence and uniqueness of solution for system (3) by using the classical Faedo-Galerkin method.

Theorem 2. *Let* (v, p) *be a solution of the system (3), then there exist positive constant* $\gamma_1 > 0$ *satisfies*

$$\mathcal{L}'(t) \le -\gamma_1 (E(t) + K_4(t)). \tag{5}$$

Theorem 3. *Let* (v, p) *solution of system (3) and* (H_1)*-*(H_2) *hold, then there exist two positive constants* s *and* η*, such that*

$$E(T) \le s e^{-\eta t}, \qquad \forall t\ \ge 0. \tag{6}$$

The goal of Theorems 2 and 3 is to prove the exponential decay of the solution towards the equilibrium point, which is zero in this case

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