On Controllability of Linear Control Delay Systems

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This research work presents the controllability properties of linear abstract delay differential control systems in Hilbert space. Many real life phenomena experience the effects of hereditary information which acts as feedback to the system. Therefore, the mathematical models of such phenomena comply with delay differential equations. In this work, the existence of solution of the considered control system is obtained by assuming that the semigroup of operators generated by the system operator of the associated uncontrolled and non-delay system. Then, various concepts of controllability are defined in respect of linear control delay systems. The main result on controllability is established under the assumption that the corresponding linear control system without delay is controllable. This work is extension of the research work by Henríquez and Prokopczyk [1] in which time-varying abstract differential equation with distributed delay in the state variable is considered.

Let X and U be Hilbert spaces of state and control variables. Consider the linear control delay system.

$$\begin{cases} \dot{x}(t) = Ax(t) + Lx(t-h) + Bu(t) + B_1u(t-h), & t \in [0,T] \\ x(\theta) = \phi(\theta), \;\; \theta \in [-h,0], \;\; h > 0, \end{cases} \tag{1}$$

where $A : D(A) \subset X \to X$ is closed linear operator; $L : X \to X$ and $B, B_1 : U \to X$ are bounded linear operators; $\phi \in C([-h,0];X)$ is initial condition.

Let us suppose that A generates a C_0-semigroup $\{S(t)\}_{t\geq 0}$ on X with $||S(t)|| \leq Me^{\omega T} = M_0$. Since L is bounded linear operator on X, therefore $A+L$ generates a C_0-semigroup $\{\hat{S}(t)\}_{t\geq 0}$ on X, which satisfies $\|\hat{S}(t)\| \leq M_0 e^{M_0 M_L T} = M_1$, where $M_0 = Me^{\omega T}$ and $M_L = ||L||$. If $\{S(t)\}_{t\geq 0}$ is compact, then $\{\hat{S}(t)\}_{t\geq 0}$ is compact. Then the mild solution of (1) is given as

$$x(t) = \begin{cases} \phi(t), \;\; -h \leq t \leq 0 \\ \hat{S}(t)\phi(0) + \int_0^t \hat{S}(t-s)Bu(s)ds + \int_0^t \hat{S}(t-s)B_1u(s-h)ds, \;\; t > 0. \end{cases}$$

Definition 1. *The reachable set for the system (1) is defined as* $\mathcal{R}_T(L) = \{x(T,u,\phi) \in X : x(\cdot,u,\phi) \in C([-h,T];X)$ *is the mild solution of (1) under some control* $u \in \mathrm{L}^2([0,T];U)\}$.

The linear nondelay control system associated to (1) is

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \;\; t \in [0,T], \\ x(0) = \phi(0). \end{cases} \tag{2}$$

Definition 2. *The reachable set for the system (2) is defined as* $\mathcal{R}_T(0) = \{x(T,u,\phi(0)) \in X : x(\cdot,u,\phi(0)) \in C([0,T];X)$ *is the solution of (2) for* $u \in \mathrm{L}^2([0,T];U)\}$.

There are several concepts of controllability for infinite-dimensional control systems, viz. exact controllability, approximate controllability, complete controllability and trajectory controllability. In this work, the approximate controllability on $[0,T]$ will be presented for the delay control system (1) by assuming that the nondelay system (2) is approximately controllable. The controllability concept is defined as follows:

Definition 3. *The linear nondelay control system is approximately controllable on* $[0, T]$ *if* $\overline{\mathcal{R}_T(0)} = X$ *and exactly controllable on* $[0, T]$ *if* $\mathcal{R}_T(0) = X$.

Definition 4. *The linear control delay system is approximately controllable on* $[0, T]$ *if* $\overline{\mathcal{R}_T(L)} = X$ *and exactly controllable on* $[0, T]$ *if* $\mathcal{R}_T(L) = X$.

Let us define operator $J_T : \mathrm{L}^2([0, T]; X) \to X$ by

$$J_T x = \int_0^T \hat{S}(T - s)x(s)ds$$

and the controllability map $\mathcal{B} : L^2([0, T]; U) \to L^2([0, T]; X)$ by $(\mathcal{B}u)(t) = Bu(t) + B_1 u(t - h)$, where $u(t - h) = 0$ for $t - h < 0$. Let $\ker(J_T)$ be the null space of J_T and $\mathcal{R}(\mathcal{B})$ be the range space of $\mathcal{B}$. Then, from a well-known result the space $L^2([0, T]; X)$ may be decomposed as $L^2([0, T]; X) = \ker(J_T) + \overline{\mathcal{R}(\mathcal{B})}$ with a projection operator $P : L^2([0, T]; X) \to \overline{\mathcal{R}(\mathcal{B})}$ such that $x - Px \in \ker(J_T)$. Now the main result is stated as

Theorem 1. *Let us assume that* $M_0 M_L ||P|| < 1$, $L^2([0, T]; X) = \ker(J_T) + \overline{\mathcal{R}(\mathcal{B})}$ *and the linear nondelay control system (2) is approximately controllable on* $[0, T]$*. Then the linear delay control system (1) is approximately controllable on* $[0, T]$.

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