On the asymptotic equivalence of ordinary and functional stochastic differential equations

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The asymptotic equivalence of two systems is a well-known topic in the theory of ODE. One can find a significant amount of academic literature that investigates this theme. Closely related results on the FDE are presented in work [1], while results on the Stochastic Differential Equations are given in [2]. This paper presents new results on the asymptotic equivalence of the Functional Stochastic Differential Equations system with a system of ODE.

For $h > 0$ we define a function space $C_h = C([-h, 0]; \mathbf{R}^d)$ of continuous functions with a norm $||\phi||_C = \sup_{\theta\in[-h,0]} |\phi(\theta)|$. Consider the system of ODE in the following form

$$dx = Axdt, \tag{1}$$

with the initial conditions $x(t_0) = x_0,\ \ t \geq t_0 \geq 0,\ \ x \in \mathbf{R^d}$, and A be a constant deterministic matrix. Along with system (1), we consider the system of Functional Stochastic Differential Equations

$$dy = (Ay + \int_{-h}^{0} B(t,\theta)y(t+\theta)d\theta)dt + \int_{-h}^{0} D(t,\theta)y(t+\theta)d\theta dW(t), \tag{2}$$

where $B(t,\theta), D(t,\theta)$ are continuous deterministic matrices for $t \geq 0,\ \ \theta \in [-h, 0]$, integrable with respect to θ. $W(t)$ is a Wiener process on a probability space $(\Omega, \mathbf{F}, P)$ with filtration $\{\mathcal{F}_t, t \geq 0\} \subset \mathbf{F}$, and there exist such $b(t)$ and $d(t)$

$$|| \int_{-h}^{0} B(t,\theta)\phi(\theta)d\theta|| \leq b(t)||\phi||_C,\ \ t \geq 0$$

$$|| \int_{-h}^{0} D(t,\theta)\phi(\theta)d\theta|| \leq d(t)||\phi||_C,\ \ t \geq 0$$

Definition 1. If for each solution $y(t)$ of system (2) there corresponds a solution $x(t)$ of system (1) such that

$$\lim_{t\to\infty} \mathbf{E}|x(t) - y(t)|^2 = 0,$$

then system (2) is called asymptotically mean square equivalent to system (1). In case when for each solution $y(t)$ of system (2) there corresponds a solution $x(t)$ of system (1) such that

$$P\{\lim_{t\to\infty} |x(t) - y(t)| = 0\} = 1$$

then system (2) is called asymptotically equivalent to system (1) with probability 1.

Theorem 1. *Let all solutions of system (1) be bounded on $t \in [0, \infty)$ and the following conditions hold*

$$\int_0^\infty |b(t)|dt \leq K_1 < \infty$$

$$\int_0^\infty |d(t)|^2 dt \leq K_1 < \infty \tag{3}$$

for some $K_1 > 0$*, then system (2) is asymptotically mean square equivalent to system (1). Also, if we change (3) by*

$$\int_0^{\infty} td^2(t)dt \leq K_1 < \infty$$

then system (2) is asymptotically equivalent to system (1) with probability 1.

1. K. G. Valeev, N. A. Kulesko, Family of solutions with a finite number of parameters of a system of differential equations with deviating argument, Ukrainian Mathematical Journal, 1968, 20, No 6, 637-646.
2. O. M. Stanzhyts'kyi, A. P. Krenevich, I. G. Novak, Asymptotic equivalence of linear stochastic Ito systems and oscillation of solutions of linear second-order equations, Differ. Equ., 2011, 47, No 6, 799-813.