Approximate solutions of ordinary differential equations via metaheuristic algorithm

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In science of engineering a dissimilar problems arise under the fast development of modern life which needs to find a very quick and precise solution that is generally very complex and hard to solve [2]. Hence optimization algorithms are the tool used to conquer this problem except in many heuristic approaches included in traditional optimization techniques which still insufficient. But, nature is considered as a source of inspiration for solving a various difficult problems since it usually finds the optimal solution to solve its problem [3]; as a result many researchers from many fields are interested to create a variety of metaheuristic algorithms that exploits different operators inspired by natural processes [1]. Metaheuristic algorithms produce the simplest procedure to solve an optimization problem. Its can commonly find very well solutions with less computational effort than traditional algorithms and iterative methods, it's have a high searched variables number and fast convergence speed.

The importance of this study is to adapt metaheuristic algorithm to solve mathematical problem by illustrating an efficient method to solve initial value problems with the aid of certain basic concepts of mathematics and metaheuristic optimization method, ODEs are considerated as an optimization problem. Our goal is to minimize the weighted residual function (error function) of the ODEs. The initial values of ODEs are represented as constraints for the optimization model. Generational distance metric is used for evaluation and judgment of approximate solutions versus exact solutions.

The applied ODEs are selected from electronics and electric engineering field that are IVPs arising from RL circuit consisting of a resistor and an inductor connected in series in the case of constant voltage approximately solved by means of the Bat-Inspired Algorithm (BA), which mimics the echolocation navigation system of bats in detecting and pursuing their prey [4]. Experimental results via numerical example when comparison between the exact solution and the algorithm outcomes in terms of solution quality showed that BA algorithm yields satisfactorily precise approximation of the solutions that confirm the utility and the effectiveness of the proposed method.

Let $f = f(x, y)$ be a real-valued function of two real variables defined for $a \leq x \leq b$, where a and b are finite, and for all real values of y. The equations

$$\begin{cases} y' = f(x, y), \\ y(a) = y_0 \end{cases} \tag{1}$$

are called initial-value problem (IVP); they symbolize the following problem: To find a function $y(x)$, continuous and differentiable for $x \in [a, b]$ such that $y' = f(x, y)$ from $y(a) = y_0$ for all $x \in [a, b]$. This problem possesses unique solution when: f is continuous on $[a, b] \times \mathbb{R}$, and satisfies the Lipschitz condition; it exists a real constant $k > 0$, as $\mid f(x, \theta_1) - f(x, \theta_2) \mid \leq k \mid \theta_1 - \theta_2 \mid$, for all $x \in [a, b]$ and all couple $(\theta_1, \theta_2) \in \mathbb{R} \times \mathbb{R}$.

The main idea behind the algorithm is to use the finite difference formula for the derivative and equation (1) we obtain,

$$\frac{y(x_j) - y(x_{j-1})}{h} \approx f(x_{j-1}, y(x_{j-1})),$$

thus,

$$\frac{y_j - y_{j-1}}{h} \approx f\Big(x_{j-1}, y_{j-1}\Big),$$

consequently, we have to consider the error formula:

$$\left[\frac{y_j - y_{j-1}}{h} - f\Big(x_{j-1}, y_{j-1})\Big)\right]^2.$$

The objective function, associated to $Y = (y_1, y_2, ..., y_d)$ will be:

$$hF(Y) = \sum_{j=1}^{d}\left[\frac{y_j - y_{j-1}}{h} - f\Big(x_{j-1}, y_{j-1}\Big)\right]^2.$$

After a comparison between the exact solutions, the algorithm outcomes and Euler method results; BA was found exponentially better than Euler method by giving accurate solutions with smallest amount error.

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