TRANSCRITICAL AND FLIP BIFURCATION IN DISCRETE-TIME MODEL WITH FRACTIONAL-ORDER

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Discrete-time models are more specific to describe epidemics than the continuous-time models and may exibit a richer dynamic behavior than its continous-time models. The objectives of this work are to study the dynamical behaviors of the discrete-time predator-prey of fractional order, the structure of equilibria and their local stability have been analyzed. The local bifurcation (transcritical and flip) have been discussed.

We consider the following fractional order predator-prey system

$$\begin{cases} \dfrac{d^\alpha x}{dt} = a\left(1-\dfrac{x}{k}\right)(x-m)\,x - l\left(1-e^{-cx}\right)y \\ \dfrac{d^\alpha y}{dt} = -dy + b\left(1-e^{-cx}\right)y \end{cases} \tag{1}$$

where $\dfrac{d^\alpha}{dt}$ is the Caputo fractional derivative with fractional-order α $(0 < \alpha \leq 1)$

x and y stand for densities of prey and predator, respectively. In order to represent biological populations, all the parameters are positive; that is, $a, k, m, l, d, b, c > 0$ and $c, m < k$.

System (1) has four fixed points: $E_0 = (0, 0)$, $E_1 = (m, 0)$, and $E_2\,(k, 0)$, $E_3\,(x^*, y^*)$, where

$$x^* = \frac{1}{c}\ln\left(\frac{b}{b-d}\right) \text{ and } y^* = \frac{ab}{dl}\left(1-\frac{x^*}{k}\right)(x^*-m)\,x^*.$$

Theorem 1. *The fixed point E_0 loses its stability through flip bifurcation when $h = \sqrt[\alpha]{\frac{2\Gamma(1+\alpha)}{am}}$ or $h = \sqrt[\alpha]{\frac{2\Gamma(1+\alpha)}{d}}$.*

Theorem 2. *The fixed point E_1 loses its stability through :*
Transcritical bifurcation when $d = b\left(1-e^{-cm}\right)$.
Flip bifurcation when $d > b\left(1-e^{-cm}\right)$ and $h = \sqrt[\alpha]{\frac{2\Gamma(1+\alpha)}{d-b(1-e^{-cm})}}$.

Theorem 3. *The fixed point E_2 loses its stability via*
Transcritical bifurcation when $d = b\left(1-e^{-ck}\right)$.
Flip bifurcation when $h = \sqrt[\alpha]{\frac{2\Gamma(1+\alpha)}{a(k-m)}}$ or $\left(d > b\left(1-e^{-cm}\right) \text{ and } h = \sqrt[\alpha]{\frac{2\Gamma(1+\alpha)}{d-b\left(1-e^{-ck}\right)}}\right)$.

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