On the stability of a type III thermoelastic Bresse system with distributed delay-time

Karek Widad[1], **Bouzettouta Lamine**[2]

[1]Laboratory of Applied Mathematics and History and Didactics of Mathematics (LAMAHIS), University of 20 August 1955, Skikda, Algeria

[2]Laboratory of Applied Mathematics and History and Didactics of Mathematics (LAMAHIS), University of 20 August 1955, Skikda, Algeria

karekwidad21@gmail.com , lami_ 750000@yahoo.fr

In this work, we present the well-posedness and the exponential stability result without the usual assumption on the wave speeds of a Bresse-type system of thermoelasticity of type III by using the semigroup method and the energy method.

In the present paper we are concerned at the Bresse system with a distributed delay term,

$$\begin{cases} \rho_1\phi_{tt} - k(\phi_x + lw + \psi)_x - k_0 l(w_x - l\phi) + \mu_1\phi_t + \int_{\tau_1}^{\tau_2} \phi_t(x, t-s)ds = 0 \\ \rho_2\psi_{tt} - b\psi_{xx} + k(\phi_x + lw + \psi) + \beta\theta_{tx} = 0 \\ \rho_1 w_{tt} - k_0(w_x - l\phi)_x + kl(\phi_x + lw + \psi) = 0 \\ \rho_3\theta_{tt} - \delta\theta_{xx} + \beta\phi_{ttx} - k\theta_{ttx} = 0. \end{cases} \tag{1}$$

where $(x, t) \in (0, 1) \times \mathbb{R}_+$ with the following boundary conditions:

$$\phi(0, t) = \phi(1, t) = \psi_x(0, t) = \psi_x(1, t) = w_x(0, t) = w_x(1, t) = \theta(0, t) = \theta(1, t) = 0, \quad t > 0 \tag{2}$$

and the initial conditions

$$\begin{cases} \phi(0, t) = \phi_0(x),\ \phi_t(0, t) = \phi_1(x),\ \psi(0, t) = \psi_0(x), \\ \psi_t(x, 0) = \psi_1(x),\ w(x, 0) = w_0(x),\ w_t(0, 1) = w_1(t), \\ \theta(x, 0) = \theta_0(x),\ \theta_t(x, 0) = \theta_1(x), \\ \phi_t(x, -\tau) = f(x, t), \quad \text{in} \quad 0 < t \le \tau_2 \\ \phi(0, t) = \psi_x(o, t) = w_x(0, t) = \theta_0(0, t) = 0, \quad t > 0 \\ \phi_x(1, t) = \psi(1, t) = w(1, t) = 0, \quad t > 0. \end{cases} \tag{3}$$

τ_1 and τ_2 are two real numbers with $0 \le \tau_1 < \tau_2, \mu_1 > 0$ is a positive constant, $\mu_2 : [\tau_1, \tau_2] \longrightarrow \mathbb{R}$ is an L^∞ function, $\mu_2 > 0$ almost everywhere, and the initial data $(\phi_0, \phi_1, \psi_0, \psi_1, w_0, w_1, \theta_0, \theta_1, f_0)$. belong to a suitable space (see below) And under the assumption

$$\mu_1 \ge \int_{\tau_1}^{\tau_2} \mu_2(s)ds \tag{4}$$

The aim of this paper is to study the well-posedness and asymptotic stability of system(1)-(3). Now we first prove the existence and uniqueness of regular solutions to problem (1)-(3) by using a semigroup theory as in [3], and Introduce the following new variable [1-2]. In order to exhibit the dissipative nature of (1), we differentiate the first, the second and the third equations of system (1) with respect to t and introduce new dependent variables

$$\varPhi = \phi_t, \varPsi = \psi_t, \mathbf{w} = w_t \text{ and } z(x, \rho, s, t) = \varPhi_t(x, t - \rho s)$$

$$z(x, \rho, s, t) = \varPhi_t(x, t - \rho s) \quad \text{in } (0, 1) \times (0, 1) \times (\tau_1, \tau_2) \times (0, \infty). \tag{5}$$

Then, we have

$$sz_t(x,\rho,s,t) + z_\rho(x,\rho,s,t) = 0 \quad \text{in } (0,1)\times(0,1)\times(\tau_1,\tau_2)\times(0,\infty).$$

Therefore, problem (1.2) takes the form

$$\begin{cases} \rho_1\Phi_{tt} - k(\Phi_x + l\mathbf{w} + \Psi)_x - k_0 l(\mathbf{w}_x - l\Phi) + \mu_1\Phi_t + \int_{\tau_1}^{\tau_2}\mu_2 z(x,1,t,s)ds = 0 \\ sz_t(x,\rho,s,t) + z_\rho(x,\rho,s,t) = 0 \\ \rho_2\Psi_{tt} - b\Psi_{xx} + k(\Phi_x + l\mathbf{w} + \Psi) + \beta\theta_{tx} = 0 \\ \rho_1\mathbf{w}_{tt} - k_0(\mathbf{w}_x - l\Phi)_x + kl(\Phi_x + l\mathbf{w} + \Psi) = 0 \\ \rho_3\theta_{tt} - \delta\theta_{xx} + \beta\Psi_{tx} - k\theta_{ttx} = 0. \end{cases} \tag{6}$$

With the initial and boundary conditions:

$$\Phi(0,t) = \Phi(1,t) = \Psi(0,t) = \Psi(1,t) = \mathbf{w}(0,t) = \mathbf{w}(1,t) = 0, \quad t > 0 \tag{7}$$

$$\begin{cases} \Phi(0,t) = \Phi_0(x),\ \Phi_t(0,t) = \Phi_1(x),\ \Psi(0,t) = \Psi_0(x), \\ \Psi_t(x,0) = \Psi_1(x),\ \mathbf{w}(x,0) = \mathbf{w}_0(x),\ \mathbf{w}_t(0,1) = \mathbf{w}_1(t), x \in (0,1) \\ \theta(x,0) = \theta_0(x),\ \theta_t(x,0) = \theta_1(x), \\ z(x,0,t,s) = \Phi_t(x,t) \text{ on } (0,1)\times(0,\infty)\times(\tau_1,\tau_2). \\ z(x,\rho,0,s) = f_0(x,\rho,s)\Phi_t(x,-\tau) = f(x,t), \quad \text{in } \ 0 < t \le \tau_2 \\ \Phi(0,t) = \Psi_x(0,t) = \mathbf{w}_x(0,t) = \theta_0(0,t) = 0, \quad t > 0 \\ \Phi_x(1,t) = \Psi(1,t) = \mathbf{w}(1,t) = 0, \quad t > 0. \end{cases} \tag{8}$$

The functional energy of solution of problem (6)-(8) is defined by

$$\begin{aligned} E(t) &= \frac{1}{2}\int_0^1 [\rho_1\Phi^2 + \rho_2\Psi^2 + \rho_1\mathbf{w}^2 + b\Psi_x^2 + \rho_3\theta_t^2 + \delta\theta_x^2 + k(\Phi_x + \Psi + l\mathbf{w})^2 + k_0(\mathbf{w}_x - l\Phi)^2]dx \\ &+ \frac{1}{2}\int_0^1\int_0^1\int_{\tau_1}^{\tau_2} s\mu_2(s)z^2(x,\rho,s,t)dsd\rho dx. \end{aligned} \tag{9}$$

Theorem 1. *Let $(\Phi,\Psi,\boldsymbol{w}\subset,\theta,z)$ be the solution of (6)-(8) Then there two positive constants α and γ such that $E(t) \le E(0)e^{-\gamma t}, \quad t > 0$.*

Lemma 1. *Let$(\Phi,\Psi,\boldsymbol{w},\theta,z)$ be the solution of (6)-(8) and assume (4) holds. Then the energy functional, defined by (9) satisfies,*

$$\frac{d}{dt}E(t) \le -r_0\int_0^1 \Phi_t^2 dt - k\int_0^1 \theta_{tx}^2 dx$$

with $r_0 = \mu_1 - \int_{\tau_1}^{\tau_2}\mu_2(s)ds$

Acknowledgements. We would also like to show our gratitude to the (Prof BOUZETTOUTA Lamine university of skikda) for sharing their pearls of wisdom with us during the course of this research.

1. N. Serge., P. Cristina. Stabilization of the wave equation with boundary or internal distributed delay Differential and Integral Equations, 2008, 21, No 9–10, 935–958.
2. S.H. Belkacem., L. Yamina. A stability result of a Timoshenko system with a delay term in the internal feedback. Applied Mathematics and Computation, 2010, 217, No 6, 2857–2869.
3. A. Pazy. Semigroups of linear operators and applications to partial differential equations. — New York Applied Math: Sciences, Springer-Verlag, 1983, 44.