INTERNATIONAL CONFERENCE OF YOUNG MATHEMATICIANS
THE INSTITUTE OF MATHEMATICS OF THE NATIONAL ACADEMY OF SCIENCES OF UKRAINE
JUNE 1B–3, 2023, KYIV, UKRAINE

SOLUTION OF CONDITIONALLY WELL-POSED PROBLEMS FOR MATRIX FACTORIZATION OF THE HELMHOLTZ EQUATION

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It is known that the Cauchy problem for elliptic equations, as well as for matrix factorizations of the Helmholtz equation belongs to the family of ill-posed problems: the solution of the problem is unique, but unstable. In unstable problems, the image of the operator is not closed, therefore, the solvability condition cannot be written in terms of continuous linear functionals. So, in the Cauchy problem for elliptic equations with data on a part of the boundary of a domain, the solution is usually unique, the problem is solvable for an everywhere dense data set, but this set is not closed. Consequently, the theory of solvability of such problems is much more difficult and deeper than the theory of solvability of the Fredholm equations. The first results in this direction appeared only in the mid-1980s in the works of L.A. Aizenberg, A.M. Kytmanov and N.N. Tarkhanov [1].

Such problems naturally arise in mathematical physics with in various fields of natural science (for example, electro-geological exploration, cardiology, electrodynamics, etc.). In general, the theory of illposed problems for elliptic systems of equations has been sufficiently formed using the workings of A.N. Tikhonov, V.K. Ivanov, Sh. Yarmukhamedov, N.N. Tarkhanov and many other famous mathematicians (see, for instance [1]). Among them, the most important ones are the so-called conditionally well-posed problems, characterized by stability in the presence of additional information about the nature of the problem data for applications. One of the most effective ways to study such problems is to construct regularizing operators.

In this paper, we are talking about solving the ill-posed Cauchy problem for matrix factorizations of the Helmholtz equation in bounded and unbounded domains of various dimensions. It is assumed that the solution of the problem exists and also it is continuously differentiable in a closed domain with exactly given Cauchy data. In this case, an explicit formula for the continuation of the solution as well as a regularization formula are established, under the indicated conditions. Their continuous approximations with a given error in the uniform metric are given instead of the Cauchy data. Additionally a stability estimate is obtained for the solution of the Cauchy problem in the classical sense (see, for instance [2-26]).

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